

ANSWER KEY

SAY/IMP
SECOND YEAR HIGHER SECONDARY EXAMINATION June - 2023

PART-~~VII~~/III

SUBJECT: MATHEMATICS (HI-80) Science

CODE NO: S-2275

VERSION: _____

80 SCORES

2½ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1		$f \circ g(x) = f(g(x))$ $= f(x^3)$ $= \cos(x^3)$	1 1 1	3
2		$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $a_{11} = 3 \quad a_{12} = 5$ $a_{21} = 4 \quad a_{22} = 6$ $A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$	1 $\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$	3
3		$\begin{vmatrix} 3 & 4 \\ 3 & 5 \end{vmatrix} = 15 - 12$ $= \underline{\underline{3}}$	2 1	3

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
4		$\text{LHL} = \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2} kx$ $= 2k$ $\text{RHL} = \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2} 4$ $= 4$ $\text{LHL} = \text{RHL}$ $2k = 4$ $k = 2$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	3
5		$3x + 4y = \sin x$ <p>Diff. w.r. to x</p> $3 + 4 \frac{dy}{dx} = \cos x$ $4 \frac{dy}{dx} = \cos x - 3$ $\frac{dy}{dx} = \frac{\cos x - 3}{4}$	 	3
6		$\frac{dr}{dt} = 5 \text{ cm/s}$ $A = \pi r^2$ $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ $= 10\pi r$ <p>when $r = 10 \text{ cm}$, $\frac{dA}{dt} = 100\pi \text{ cm}^2/\text{s}$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	3

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
10	(i)	(a) $\frac{\pi}{4}$	1	4
	(ii)	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{\pi}{3}$ $= \frac{2\pi}{3}$	1	
	(iii)	$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
11	(i)	$\frac{dy}{dt} = 2a$	1	4
	(ii)	$\frac{dx}{dt} = 2at$	1	
	(iii)	$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ $= \frac{1}{t}$	1	
12	(i)	(iv) $\sin x + C$	1	4
	(ii)	(iii) $\sin^{-1}\left(\frac{x}{a}\right) + C$	1	
	(iii)	(ii) $\tan^{-1} x + C$	1	
	(iv)	(i) $\log x^2 - 1 + C$	1	
13		Area = $\int_a^b y dx$ $= \int_1^4 \sqrt{x} dx$ $= \frac{2}{3} \left[x^{3/2} \right]_1^4$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= \frac{2}{3} \left[4^{3/2} - 1^{3/2} \right]$ $= \frac{2}{3} \times 7 = \frac{14}{3} \text{ sq. units}$	1	4
14		$\frac{dy}{1+y^2} = (1+x^2) dx$ $\int \frac{dy}{1+y^2} = \int (1+x^2) dx$ $\tan^{-1} y = x + \frac{x^3}{3} + C$	1 1 1+1	4
15	(i) (ii) (iii)	$\vec{a} + \vec{b} = 4\hat{i} + 2\hat{j} + 5\hat{k}$ $\vec{a} - \vec{b} = 2\hat{i} + 3\hat{k}$ $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{vmatrix}$ $= \hat{i}(6-0) - \hat{j}(12-10) + \hat{k}(0-4)$ $= 6\hat{i} - 2\hat{j} - 4\hat{k}$	1 1 1 1/2 1/2	4
16		$\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k} \quad \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$ $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k} \quad \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$ $\vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}$	1/2 1/2 1/2 1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= -3\hat{i} + 3\hat{k}$ $ \vec{b}_1 \times \vec{b}_2 = \sqrt{18}$ $d = \left \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{ \vec{b}_1 \times \vec{b}_2 } \right $ $= \left \frac{(-3\hat{i} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} - 2\hat{k})}{\sqrt{18}} \right = \frac{9}{\sqrt{18}}$ <p>Remarks: (i) Any alternative method with correct answer give 5.1.4 SCORES (ii) For formula in cartesian form give 1/2 score</p>	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	<p>4</p>
17		<p>Equation of plane is</p> $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$ $\begin{vmatrix} x-2 & y-5 & z+3 \\ -2-2 & -3-5 & 5+3 \\ 5-2 & 3-5 & -3+3 \end{vmatrix} = 0$ $\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$ $2x + 3y + 4z = 7$ <p>Remarks: For any alternative method with correct answer give 4 scores.</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p>	<p>4</p>

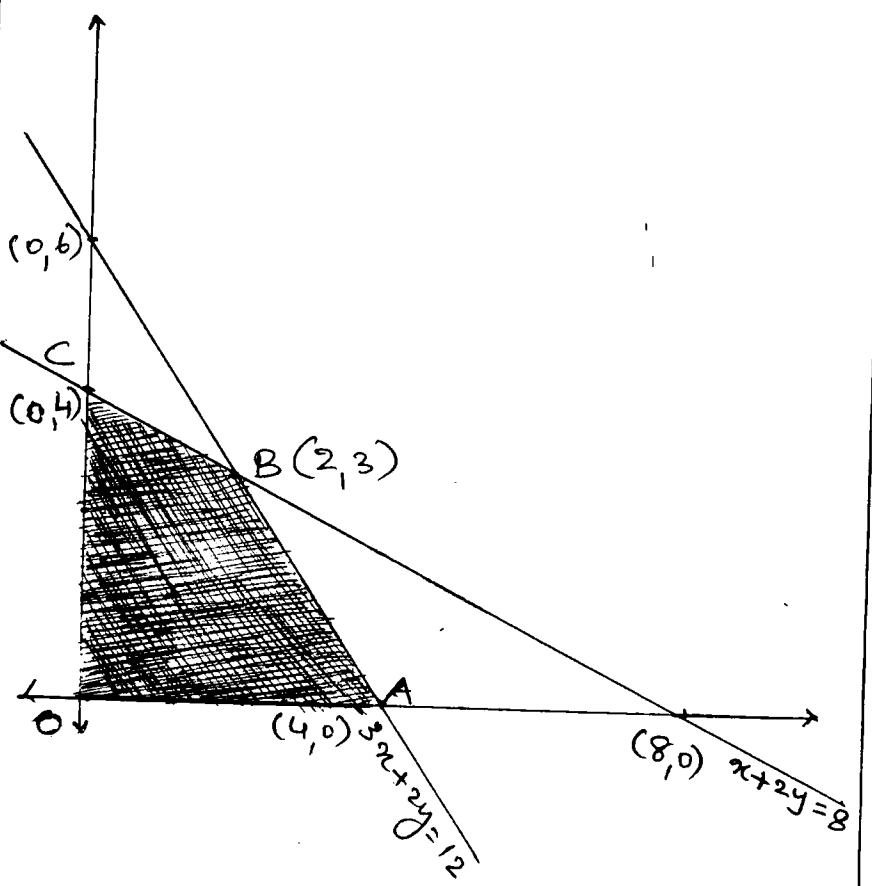
No	Sub Qns	Answer Key/Value Points	Score	Total Score
18	(i)	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= \frac{6}{11} + \frac{5}{11} - \frac{7}{11}$ $= \frac{4}{11}$	1 1/2 1/2	4
	(ii)	$P(A/B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{4/11}{5/11} = \frac{4}{5}$	1 1	
19	(i)	$AB = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ $= \begin{bmatrix} 0+4 & 3+6 \\ 0+8 & 1+12 \end{bmatrix}$ $= \begin{bmatrix} 4 & 9 \\ 8 & 13 \end{bmatrix}$	1/2 1 1/2	6
	(ii)	$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$ $P = \frac{1}{2}(A + A')$ $= \frac{1}{2} \left(\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} \right)$ $= \frac{1}{2} \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 11/2 \\ 11/2 & 7 \end{bmatrix}$ $P' = \begin{bmatrix} 1 & 11/2 \\ 11/2 & 7 \end{bmatrix} = P$	1/2 1/2 1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		<p>$\therefore P$ is symmetric matrix</p> $Q = \frac{1}{2}(A - A')$ $= \frac{1}{2}\left(\begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}\right)$ $= \frac{1}{2}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ $Q' = \begin{bmatrix} 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 \end{bmatrix} = -Q$ <p>$\therefore Q$ is skew symmetric</p> $P + Q = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 7 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$ $= \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} = A$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	
20	(i)	$10 - 12 = 5x - 6x$ $x = 2$	1	
	(ii)	$Ax = B$ $A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ $ A = -11$ $\text{adj} A = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$ $A^{-1} = \frac{1}{ A } \text{adj} A$	<p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score									
		$A^{-1} = \frac{-1}{11} \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$ $X = A^{-1}B$ $= \frac{-1}{11} \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 7 \end{bmatrix}$ $X = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$ $x = 3, y = -1$	<p>4/2</p> <p>4/2</p> <p>4/2</p> <p>4/2</p>	6									
21	(i) (a)	$\frac{dy}{dx} = 2x + 1$ <p>Slope of tangent at $x = 2$ is</p> $2 \times 2 + 1 = 5$	1	6									
	(b)	Slope of normal at $x = 2$ is $-\frac{1}{5}$	1										
	(ii)	$f'(x) = 2x + 2$ $f'(x) = 0 \Rightarrow x = -1$ <p>Divide the real line into disjoint intervals such as $(-\infty, -1)$ and $(-1, \infty)$</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>Interval</th> <th>Sign of $f'(x)$</th> <th>Nature of $f(x)$</th> </tr> </thead> <tbody> <tr> <td>$(-\infty, -1)$</td> <td>< 0</td> <td>strictly decreasing</td> </tr> <tr> <td>$(-1, \infty)$</td> <td>> 0</td> <td>strictly increasing</td> </tr> </tbody> </table>	Interval		Sign of $f'(x)$	Nature of $f(x)$	$(-\infty, -1)$	< 0	strictly decreasing	$(-1, \infty)$	> 0	strictly increasing	1
Interval	Sign of $f'(x)$	Nature of $f(x)$											
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			4/2										
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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22	(i)	$\int \frac{2x}{\sqrt{9+x^2}} dx$ <p>Put $9+x^2 = t$ $2x dx = dt$</p> $\int \frac{2x}{\sqrt{9+x^2}} dx = \int \frac{1}{\sqrt{t}} dt$ $= 2\sqrt{t} + C$ $= 2\sqrt{9+x^2} + C$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1</p> <p>1/2</p>	
	(ii)	$I = \int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \quad \text{--- (1)}$ <p>By P₄: $I = \int_0^{\pi/2} \frac{\cos^5(\pi/2 - x)}{\sin^5(\pi/2 - x) + \cos^5(\pi/2 - x)} dx$</p> $I = \int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx \quad \text{--- (2)}$ $2I = \int_0^{\pi/2} dx$ $= [x]_0^{\pi/2}$ $= \pi/2$ $I = \pi/4$	<p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p> <p>1/2</p>	<p>6</p>

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score												
23	(i)	$ \vec{a} = 3$ $\hat{a} = \frac{\vec{a}}{ \vec{a} }$ $= \frac{i^{\wedge} + 2j^{\wedge} + 2k^{\wedge}}{3} = \frac{1}{3}i^{\wedge} + \frac{2}{3}j^{\wedge} + \frac{2}{3}k^{\wedge}$	1 1 1	6												
	(ii)	$\vec{a} \cdot \vec{b} = 60$ $ \vec{b} = \sqrt{114}$ <p>Projection of \vec{a} on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ \vec{b} }$</p> $= \frac{60}{\sqrt{114}}$	1 1 4/2 4/2													
24		$x + 2y = 8$ <table border="1" data-bbox="322 1288 710 1473"> <tr> <td>x</td> <td>0</td> <td>8</td> </tr> <tr> <td>y</td> <td>4</td> <td>0</td> </tr> </table> $3x + 2y = 12$ <table border="1" data-bbox="316 1691 710 1886"> <tr> <td>x</td> <td>0</td> <td>4</td> </tr> <tr> <td>y</td> <td>6</td> <td>0</td> </tr> </table>	x	0	8	y	4	0	x	0	4	y	6	0	4/2 4/2	
x	0	8														
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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score										
		 <p>Point B is (2,3)</p>	3	6										
		<table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 35%;">Corners</th> <th style="width: 65%;">Z = -3x + 4y</th> </tr> </thead> <tbody> <tr> <td>(0,0)</td> <td style="text-align: center;">0</td> </tr> <tr> <td>(4,0)</td> <td style="text-align: center;">-12</td> </tr> <tr> <td>(2,3)</td> <td style="text-align: center;">6</td> </tr> <tr> <td>(0,4)</td> <td style="text-align: center;">16</td> </tr> </tbody> </table>	Corners	Z = -3x + 4y	(0,0)	0	(4,0)	-12	(2,3)	6	(0,4)	16	1/2	
Corners	Z = -3x + 4y													
(0,0)	0													
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(0,4)	16													
		<p>Minimum value of Z = -12 at (4,0)</p> <p>Remarks:-</p> <ul style="list-style-type: none"> (i) For x axis and y axis give 1/2 score (ii) For each correct line give 1 score each. (iii) For correct shading 1/2 score (iv) For any 3 correct corner points - 1 1/2 score 	1/2											

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
25	(i)	$\sum_{i=1}^n P_x = 1$ $k + 2k + 3k + 2k = 1$ $k = \frac{1}{8}$	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	6
	(ii)	$P(x < 3) = P(x=1) + P(x=2)$ $= k + 2k = 3k$ $= \frac{3}{8}$	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	
	(iii)	$P(x \geq 3) = P(x=3) + P(x=4) + P(x=5)$ $= 3k + 2k = 5k$ $= \frac{5}{8}$ <p>Remark :- In (iii), for any alternative method with correct answer give score 2.</p>	<p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p>	