ANSWER KEY

Second YEAR HIGHER SECONDARY SAY/IMP EXAMINATION ~~March~~ JUNE 2023

PART-I/II/III

SUBJECT: Mathematics Science - 60

CODE NO: S 2227 VERSION:

60 SCORES 2 HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.		$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}_{2\times 3}$	1	3
		$a_{11} = 2\times1+1 = 3$ $\quad a_{12} = 2\times1+2 = 4$ $a_{13} = 2\times1+3 = 5$ $\quad a_{21} = 2\times2+1 = 5$ $a_{22} = 2\times2+2 = 6$ $\quad a_{23} = 2\times2+3 = 7$	1½	
		$A = \begin{bmatrix} 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}_{2\times 3}$	$\frac{1}{2}$	
2.		Assume $f(x_1) = f(x_2)$ $3+4x_1 = 3+4x_2$ $4x_1 = 4x_2$ $x_1 = x_2$ f is one-one	1	
		Let $y = f(x) \in R$ $y = 3+4x$ $4x = y-3$ $x = \frac{y-3}{4} \in R$ $\therefore f$ is onto	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		f is one-one and onto $\therefore$ f is bijective	1	3
3		Area, $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$ $= \frac{1}{2}[3(2-1) - 8(-4-5) + 1(-4-10)]$ $= \frac{1}{2}[3 + 72 - 14]$ $= \frac{61}{2}$ sq. units	1 1 1	3
4.		$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x+2 = 1+2 = 3$ $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x-2 = 1-2 = -1$ $\lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$ $\therefore \lim_{x \to 1} f(x)$ does not exist $\therefore$ f is discontinuous at $x=1$	1 1 1	3

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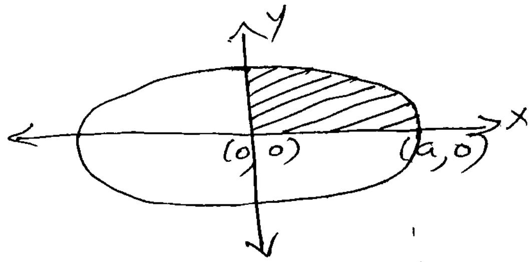
5		$f(x) = x^2 - 4x + 6$		
		$f'(x) = 2x - 4$	1	
		$f'(x) = 0 \Rightarrow 2x - 4 = 0$		
		$\Rightarrow 2x = 4$		
		$\Rightarrow x = 2$	1	3
		Consider the intervals $(-\infty, 2)$ and $(2, \infty)$		
		In $(-\infty, 2)$, $f'(x) < 0$		
		In $(2, \infty)$, $f'(x) > 0$	1	
		$\therefore$ f is strictly increasing in $(2, \infty)$		
		f is strictly decreasing in $(-\infty, 2)$		
6	(i)	$P(A) = 0.3$, $P(B) = 0.6$		
		$P(A \cap B) = P(A) \cdot P(B)$	1	
		$P(A \cap B) = 0.3 \times 0.6$		
		$= 0.18$	½	3
	(ii)	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	1	
		$P(A \cup B) = 0.3 + 0.6 - 0.18$	½	
		$= 0.72$		

C

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
7		$\vec{r} = \vec{a} + \lambda\vec{b}$	1	3
		$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$	½	
		$\vec{b} = 3\vec{i} + 2\vec{j} - 2\vec{k}$	½	
		$\vec{r} = (\vec{i} + 2\vec{j} + 3\vec{k}) + \lambda(3\vec{i} + 2\vec{j} - 2\vec{k})$	1	
8.		$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\lvert\vec{a}\rvert\,\lvert\vec{b}\rvert}$	1	3
		$\cos\theta = \dfrac{(\vec{i} - 2\vec{j} + 3\vec{k})\cdot(3\vec{i} - 2\vec{j} + \vec{k})}{\sqrt{1+4+9}\;\sqrt{9+4+1}}$	1	
		$\cos\theta = \dfrac{1\times3 + -2\times-2 + 3\times1}{\sqrt{14}\;\sqrt{14}}$		
		$\cos\theta = \dfrac{3+4+3}{14}$		
		$\cos\theta = \dfrac{10}{14} = \dfrac{5}{7}$		
		$\theta = \cos^{-1}(5/7)$	1	
9	(i)	$\vec{a} + \vec{b} = 4\vec{i} + 4\vec{j}$	½	4
	(ii)	$\vec{a} - \vec{b} = 2\vec{i} + 4\vec{k}$	½	
	(iii)	Unit vector $\perp$ar to $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$		
		$= \dfrac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{\lvert(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\rvert}$	1	
		$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} i & j & k \\ 4 & 4 & 0 \\ 2 & 0 & 4 \end{vmatrix} = 16\vec{i} - 16\vec{j} - 8\vec{k}$	1	
		$\lvert(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\rvert = 24$	½	
		unit vector $= \dfrac{16\vec{i} - 16\vec{j} - 8\vec{k}}{24}$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
10	(i)	$(2,2)$	1	
	(ii)	$R=\{(1,3),(2,6),(3,9),(4,12)\}$	$1\frac{1}{2}$	
		R is not reflexive $\because (1,1)\notin R$	$\frac{1}{2}$	4
		R is not symmetric $\because (1,3)\in R$ but $(3,1)\notin R$	$\frac{1}{2}$	
		R is not transitive $\because (1,3)\in R,\ (3,9)\in R$ but $(1,9)\notin R$	$\frac{1}{2}$	
11	(i)	2	1	
	(ii)	$\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$		
		$\frac{dy}{1+y^2}=\frac{dx}{1+x^2}$ variables are separated	1	4
		$\int \frac{dy}{1+y^2}=\int\frac{dx}{1+x^2}$	1	
		$\tan^{-1}y=\tan^{-1}x+C$	1	

C

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
12		 Area enclosed by the ellipse $= 4 \times$ Area in I^{st} quadrant $= 4\int_0^a y\,dx$ $= 4 \times \int_0^a \frac{b}{a}\sqrt{a^2-x^2}\,dx$ $= \frac{4b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$ $= \frac{4b}{a}\left[\frac{\pi a^2}{2}\right]$ $= \pi ab$ sq.units Remark : Drawing figure only, give 1 score	1 1 1 1	4
13		$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, $A^T = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$ $A + A^T = \begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$ $\frac{1}{2}(A+A^T) = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix}$ $= \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}$ $= P$ is symmetric $A - A^T = \begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix}$ $\frac{1}{2}(A-A^T) = \begin{bmatrix} 0 & -1/2 & -5/2 \\ 1/2 & 0 & 3 \\ 5/2 & -3 & 0 \end{bmatrix} = Q$ is SkewSymmetric	1 ½ ½ ½ ½ ½	4

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$A = P + Q$, where P is symmetric and Q is skew symmetric	½	
14		$S.D = \left\lvert \dfrac{(\bar{b}_1 \times \bar{b}_2)\cdot(\bar{a}_2 - \bar{a}_1)}{\lvert \bar{b}_1 \times \bar{b}_2 \rvert} \right\rvert$	1	
		$\vec{a}_1 = \vec{i} + \vec{j}$ $\quad \vec{b}_1 = 2\vec{i} - \vec{j} + \vec{k}$ $\vec{a}_2 = 2\vec{i} + \vec{j} - \vec{k}$ $\quad \vec{b}_2 = 3\vec{i} - 5\vec{j} + 2\vec{k}$	½	
		$\vec{a}_2 - \vec{a}_1 = \vec{i} - \vec{k}$	½	
		$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\vec{i} - \vec{j} - 7\vec{k}$	1	4
		$\lvert \vec{b}_1 \times \vec{b}_2 \rvert = \sqrt{59}$	½	
		$(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1) = 10$	½	
		$S.D = \left\lvert \dfrac{10}{\sqrt{59}} \right\rvert = \dfrac{10}{\sqrt{59}}$		
15		E_1 : bag I is selected E_2 : bag II is selected A : drawing a red ball		
		$P(A/E_1) = \frac{3}{8}$ $P(A/E_2) = \frac{2}{8}$	1	
		$P(E_1) = P(E_2) = \frac{1}{2}$	1	4
		$P(E_1/A) = \dfrac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$	1	
		$= \dfrac{\frac{1}{2} \times \frac{3}{8}}{\frac{1}{2} \times \frac{3}{8} + \frac{1}{2} \times \frac{2}{8}} = \dfrac{3/16}{5/16} = \dfrac{3}{5}$	1	

C

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
16	(i)	$\frac{\pi}{6}$	1	
	(ii)	$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	1	
	(iii)	$\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$		4
		$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right)$	1	
		$= \sin^{-1}\left(\sin \frac{\pi}{3}\right)$	½	
		$= \frac{\pi}{3}$	½	
17	(i)	$\left[x^2+3x\right]_0^1 = 4$	1	
	(ii)	$\int \frac{1 \, dx}{x^2+2x+1^2-1^2+2}$	1	
		$= \int \frac{1 \, dx}{(x+1)^2+1}$	½	
		$= \tan^{-1}(x+1) + C$	½	
	(iii)	Let $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$ —— (1)	½	6
		Also $I = \int_0^{\pi/2} \frac{\sqrt{\cos(\pi/2 - x)}}{\sqrt{\cos(\pi/2-x)}+\sqrt{\sin(\pi/2-x)}} dx$	½	
		$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ —— (2)	½	
		(1) + (2) $2I = \int_0^{\pi/2} \frac{\sqrt{\cos x}+\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$	½	
		$2I = \int_0^{\pi/2} 1 \, dx = [x]_0^{\pi/2} = \pi/2$	½	
		$I = \pi/4$	½	

C

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
18		The given system can be expressed as $AX=B$, where	1	
		$A=\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B=\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$		
		$\lvert A\rvert = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix} = 2(-4+4)+3(-6+4)+5(3-2)=-1\neq 0$	1	
		A^{-1} exists		
		$A^{-1}=\frac{\text{adj }A}{\lvert A\rvert}$	1	
		$\text{adj }A=\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$	1	6
		$A^{-1}=\frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$		
		$=\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$	1	
		$X=A^{-1}B$		
		$\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}=\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	1	
		$x=1,\ y=2,\ z=3$		

C

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
19	(i)	$x^2 + xy + y^2 = 100$		
		Differentiating w.r.t x		
		$2x + x\frac{dy}{dx} + y \cdot 1 + 2y \cdot \frac{dy}{dx} = 0$	1	
		$\frac{dy}{dx}(x+2y) = -(2x+y)$	1	
		$\frac{dy}{dx} = \frac{-(2x+y)}{x+2y}$	1	6
	(ii)	$A = \pi r^2$	1	
		Differentiating w.r.t r		
		$\frac{dA}{dr} = 2\pi r$	1	
		When $r = 5$ cm		
		$\frac{dA}{dr} = 2\pi \times 5$	1	
		$= 10\pi\ cm^2/cm$		

C

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
20		$x+y=50$ $2x+y=80$	1	6

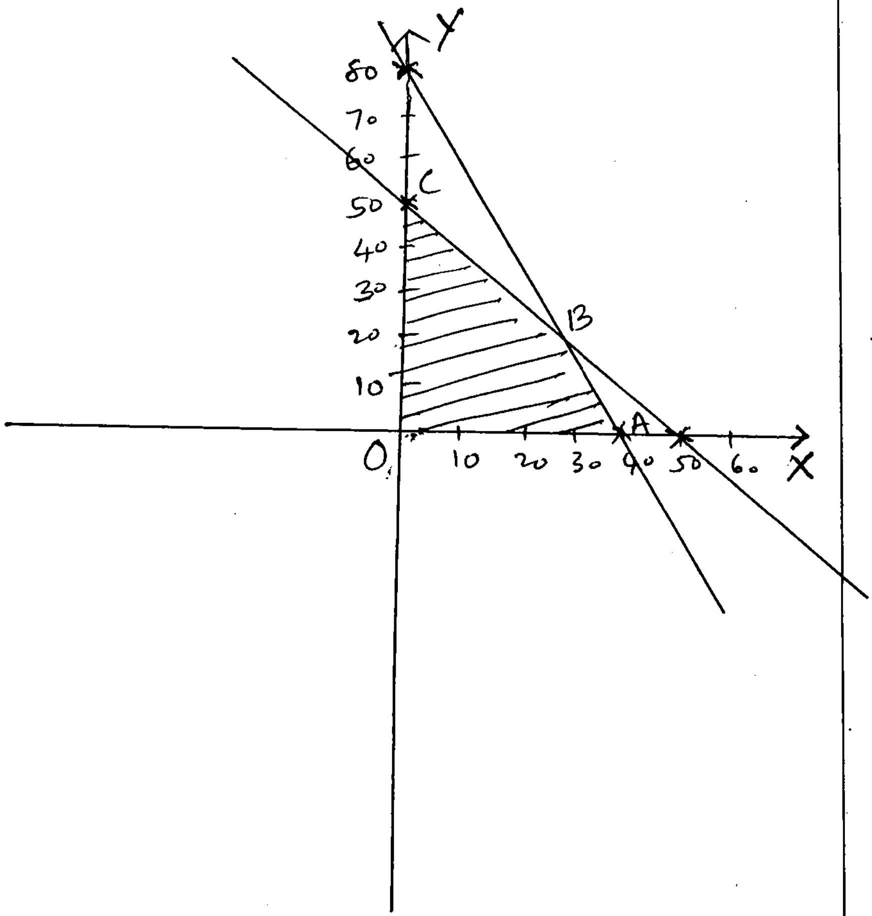
$x+y=50$

x	0	50
y	50	0

$2x+y=80$

x	0	40
y	80	0

1



3

Corner Pts	$Z=5x+2y$
A(40,0)	200 → Maximum
O(0,0)	0
C(0,50)	100
B(30,20)	190

2

Maximum Value is 200

at A(40,0)

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