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DITTO WS

ANSWER KEY

SECOND YEAR HIGHER SECONDARY SAY/Imp EXAMINATION June 2023

PART-III

SUBJECT: MATHEMATICS

CODE NO: S-2254

VERSION:

80 SCORES

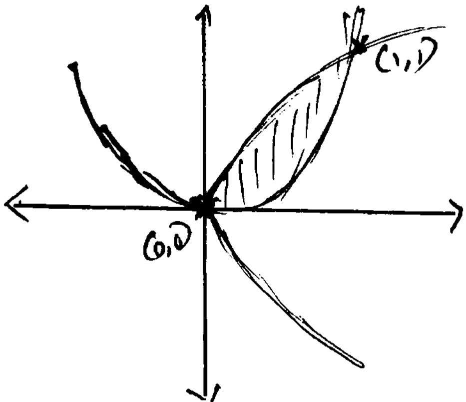
2 1/2 HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1	a	(b) 2×3	1	3
	b	$x = 1$ $2y = 4 \Rightarrow y = 2$ $z - 1 = 3 \Rightarrow z = 4$.	2	
2		$(1,1)\ (2,2)\ (3,3)\ (4,4) \in R$ Hence R is reflexive.	1	3
		$(1,2) \in R$ but $(2,1) \notin R$ Hence R is not symmetric	1	
		$(a,b) \in R$, $(b,c) \in R$ and $(a,c) \in R$ Hence R is transitive	1	
3	(a)	(c) $k^5 \lvert A \rvert$	1	3
	(b)	Area of the triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $= \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 1 \end{vmatrix}$	1	
		$= \frac{1}{2} \times 2 = 1$ sq. unit	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
4	(a)	(a) 1	1	3
	(b)	$2 + 3\frac{dy}{dx} = \cos x$	1	
		$\frac{dy}{dx} = \frac{\cos x - 2}{3}$	1	
5		$\frac{dy}{dx} = -5\sin x - 3\cos x.$	1	3
		$\frac{d^2y}{dx^2} = -5\cos x + 3\sin x.$	1	
		$= -y$	1	
		$\frac{d^2y}{dx^2} + y = 0.$		
6	(a)	(b) $\log x.$	1	3
	(b)	$A = \pi r^2$	$1/2$	
		$\frac{dA}{dr} = 2\pi r$	1	
		when $r = 5cm$, $\frac{dA}{dr} = 2\pi \times 5 = 10\pi$	$1/2$	
7	(a)	(b) 2.	1	
	(b)	$x^2\frac{dy}{dx} = 2xy.$		3
		$\frac{1}{y}dy = \frac{2}{x}dx.$		
		$\int \frac{1}{y}dy = 2\int \frac{1}{x}dx.$	1	
		$\log\lvert y\rvert = 2\log\lvert x\rvert + \log\lvert c\rvert$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
8		$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ $\quad \vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$		
		$\vec{r} = \vec{a} + \lambda \vec{b}$	1/2	
		$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$	1	3
		Cartesian equation is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$	1/2	
		$\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$	1	
9	(a)	(d) $\vec{0}$	1	
	(b)	$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$		4
		$= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$	1	
		$= \vec{a} \times \vec{b} + \vec{a} \times \vec{c} + \vec{b} \times \vec{c} - \vec{a} \times \vec{b} - \vec{a} \times \vec{c} - \vec{b} \times \vec{c}$	1	
		$= \vec{0}$	1	
10		$\vec{a_1} = 3\hat{i} + \hat{j} - 3\hat{k}$ $\quad \vec{b_1} = 2\hat{i} + 5\hat{j} + 4\hat{k}$		
		$\vec{a_2} = -5\hat{i} - 2\hat{j} + 3\hat{k}$ $\quad \vec{b_2} = \hat{i} + \hat{j} + 2\hat{k}$	1	
		$\vec{a_2} - \vec{a_1} = -8\hat{i} - 3\hat{j} + 6\hat{k}$	1	
		$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 5 & 4 \\ 1 & 1 & 2 \end{vmatrix} = 6\hat{i} - 3\hat{k}$	1	4
		$\lvert \vec{b_1} \times \vec{b_2} \rvert = 3\sqrt{5}$		
		$S.D = \left\lvert \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{\lvert \vec{b_1} \times \vec{b_2} \rvert} \right\rvert$	1	
		$= \frac{66}{3\sqrt{5}} = \frac{22}{\sqrt{5}}$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
11	(a)	$\frac{dy}{dx} + \frac{2}{x} y = x$ $IF = e^{\int P dx} = e^{\log x^2} = x^2$	2	4
	(b)	$y \cdot e^{\int P dx} = \int Q \cdot e^{\int P dx} \cdot dx$ $y x^2 = \int x^3 dx$ $y = \frac{x^2}{4} + c$ Remark $I.F = e^{\int P dx}$ give one score Solution $y \cdot e^{\int P dx} = \int Q \cdot e^{\int P dx} dx$ give 1 score	2	
12	(a)	y	1	4
	(b)	$f \circ g(x) = f(g(x))$ $= f(x^{1/3}) = 8x$ $g \circ f(x) = g(f(x))$ $= g(8x^3) = 2x$	1/2 1 1/2 1	
13	(a)	(b) $[-\pi/2, \pi/2]$	1	4
	(b)	$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$ $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{11}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\times\frac{2}{11}}\right)$ $= \tan^{-1} 3/4$	1 1 1	

Qn No.	Sub Qns.	Answer key / Value Points.	Score	Total Score
14	(a)	$\log\lvert x\rvert + C$.	1	
	(b)	$\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} = -\int_1^0 \frac{dt}{1+t^2}$ put $\cos x = t$, $x=0,\ t=1$, $x=\pi/2,\ t=0$.	1	4.
		$= -\tan^{-1} t\Big]_1^0$	1	
		$= -\left[\tan^{-1}(0) - \tan^{-1}(1)\right]$	1	
		$= \frac{\pi}{4}$		
15			1/2	4
		Solving $y^2 = x$ and $x^2 = y \Rightarrow x=0$ or $x=1$.	1	
		Required Area $= \int_0^1 \sqrt{x}\,dx - \int_0^1 x^2\,dx$.	1	
		$= \frac{2}{3}\left[x\sqrt{x}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$	1	
		$= \frac{1}{3}$ sq. units.	1/2	
16	(a)	(c) $\cos x$.	1	
	(b)	$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x) = f(5)$ Since f is continuous	1	4
		$15-8 = 2k$	1	
		$2k = 7$	1	
		$k = 7/2$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
17	(a)	1	1	4
	(b)	Equations of the plane $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x-x_2 & y-y_2 & z-z_2 \\ x-x_3 & y-y_3 & z-z_3 \end{vmatrix}=0.$	1	
		$\begin{vmatrix} x-2 & y-5 & z+3 \\ -2-2 & -3-5 & 5+3 \\ 5-2 & 3-5 & -3+3 \end{vmatrix}=0.$	1	
		$16(x-2)+24(y-5)+32(z+3)=0$	1	
		$2x+3y+4z-7=0$		
18	(a)	1	1	4
	(b)	$P(A/B)=\dfrac{P(A\cap B)}{P(B)}=\dfrac{4}{9}$	1	
		$P(B/A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{4}{7}$	1	
		$P(A\cup B)=P(A)+P(B)-P(A\cap B)=\dfrac{12}{13}$	1	
19	(a)	$A=\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$	2	6
		$A'=\begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix}$		
		$P=\dfrac{A+A'}{2}=\begin{bmatrix} 1 & 3/2 & 2 \\ 3/2 & 2 & 5/2 \\ 2 & 5/2 & 3 \end{bmatrix}$ is symmetric	2	
		$Q=\dfrac{A-A'}{2}=\begin{bmatrix} 0 & -3/2 & -3 \\ \frac{3}{2} & 0 & -3/2 \\ 3 & 3/2 & 0 \end{bmatrix}$ is skew symmetric	2	
		$P+Q=A$		

70

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
20	(a)	$f'(x) = 2x-4$.	1	
		$f'(x)=0 \Rightarrow x=2$.		
		In the interval $(-\infty, 2)$ $f'(x)$ is negative	1	
		In the interval $(2, \infty)$ $f'(x)$ is positive.		
		Hence $f(x)$ is strictly decreasing in $(-\infty, 2)$	1	6
	(b)	$f'(x) = 6x^2 - 30x + 36 = 6(x-3)(x-2)$	1	
		$f'(x)=0 \Rightarrow x = 2, 3$.		
		$f(1) = 24$, $f(2) = 29$, $f(3) = 28$, $f(5) = 56$.	1	
		Absolute Max = Max $\{24, 29, 28, 56\} = 56$	1/2	
		Absolute Min = Min $\{24, 29, 28, 56\} = 24$.	1/2	
21	(i)	$\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$	1	
	(ii)	$\lvert A \rvert = 40$	1	
	(iii)	$\text{adj } A = \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$	2	6
		$A^{-1} = \dfrac{\text{adj } A}{\lvert A \rvert} = \dfrac{1}{40} \begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$	1	
		$X = A^{-1} B = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$	1	
		$x = 1$, $y = 2$, $z = -1$.		
22	a	$\vec{a} + \vec{b} = 3\hat{j} + 4\hat{k}$	1	
		$\vec{a} - \vec{b} = -\hat{i} - 2\hat{j} - 2\hat{k}$	1	
		$\vec{a} \cdot \vec{b} = 4$	1	

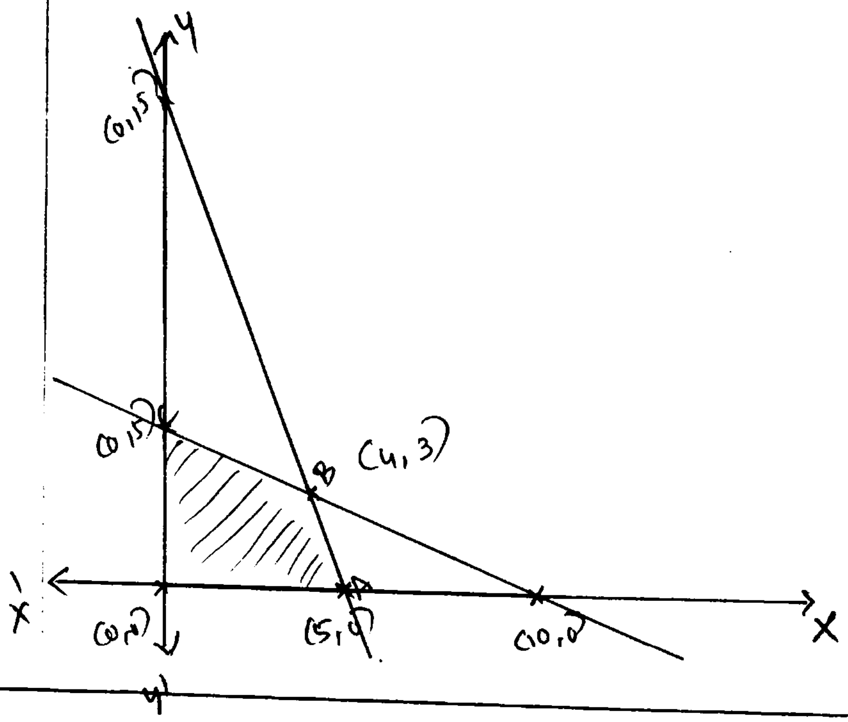
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22	b	$(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = 8\hat{i}+2\hat{j}-6\hat{k}$	1	
	c	$\left\|(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})\right\| = \sqrt{104} = 2\sqrt{26}$	1	6
		Unit vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ is $\dfrac{(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})}{\left\|(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})\right\|} = \dfrac{1}{\sqrt{26}}(4\hat{i}+\hat{j}-3\hat{k})$	1	
23	a	$P(E\cap F) = P(E)+P(F)-P(E\cup F) = 0.12$. $P(E)\cdot P(F) = 0.6\times 0.2 = 0.12$ $P(E\cap F) = P(E)\,P(F)$. E and F are independent.	2	
	b	E_1: Bag I is selected E_2: Bag II is selected. $P(E_1) = P(E_2) = \frac{1}{2}$.	1	6
		A: red ball is drawn $P(A\|E_1) = \frac{5}{8}$ $\quad P(A\|E_2) = \frac{3}{10}$.	1	
		$P(E_2\|A) = \dfrac{P(E_2)\,P(A\|E_2)}{P(E_1)\,P(A\|E_1)+P(E_2)\,P(A\|E_2)}$	1	
		$= \dfrac{\frac{1}{2}\times\frac{3}{10}}{\frac{1}{2}\times\frac{5}{8}+\frac{1}{2}\times\frac{3}{10}} = \dfrac{18}{37}$	1	

Qn No	Sub Qn	Answer key / Value points	Score	Total Score
24	(a)	$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$	1/2	
		$= \frac{-1}{x+1} + \frac{2}{x+2}$	1	
		$\int \frac{x}{(x+1)(x+2)}\,dx = -\int \frac{1}{x+1}\,dx + 2\int \frac{1}{x+2}\,dx$	1/2	
		$= -\log\lvert x+1\rvert + 2\log\lvert x+2\rvert + C$	1	6
	(b)	$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$	1	
		$= x\sin x + \cos x + C$		
		$\int_0^{\pi/2} x\cos x\,dx = \left[x\sin x + \cos x\right]_0^{\pi/2}$	1	
		$= \frac{\pi}{2} - 1$	1	
25	(a)	$x + 2y = 10$ $\quad$ $3x + y = 15$		

x	0	10
y	5	0

x	0	5
y	15	0

Score: 1



Score: 3

10/10

Qn .No	Sub Qns	Answer Key | Value points	Score	Total Score
25	b	(see table below)	2	6

Corner point	Value of Z $Z = 3x + 2y$
O (0, 0)	$Z = 0$
A (5, 0)	$Z = 15$
B (4, 3)	$Z = 18$
C (0, 5)	$Z = 10$

Maximum value of Z is 18 at B(4,3)