ANSWER KEY

SECOND YEAR HIGHER SECONDARY SAY/IMP EXAMINATION June 2023

PART-I/II/III

SUBJECT: MATHEMATICS (COMMERCE -80)

CODE NO: S 2255 VERSION: ___

80 SCORES 2 ½ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.	i)	$f(x_1)=f(x_2)$ $\frac{2x_1-1}{3}=\frac{2x_2-1}{3}$ $2x_1-1=2x_2-1$ $2x_1=2x_2 \Rightarrow x_1=x_2$ $\therefore f$ is one-one	1	
		$y=\frac{2x-1}{3} \Rightarrow 3y+1=2x$ $\Rightarrow x=\frac{3y+1}{2} \in X$ $\therefore f$ is onto.	1	
	ii)	$f^{-1}(x)=\frac{3x+1}{2}$	1	3
2.	i)	B) $\|A\|^2$	1	
	ii)	$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $a_{11}=2\times1-1=1$ $a_{12}=2\times1-2=0$ $a_{21}=2\times2-1=3$ $a_{22}=2\times2-2=2$ $A=\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$	2	3
3.	i)	A) $B^{-1}A^{-1}$	1	
	ii)	$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix}=\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ $x^2-36=36-36$	1	
		$x^2-36=0$ $x^2=36$ $x=\pm 6$	1	3

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4.		$\lim_{x\to 5^-} f(x) = \lim_{x\to 5} kx+1 = 5k+1$ $\lim_{x\to 5^+} f(x) = \lim_{x\to 5} 3x-5 = 3\times5-5 = 10$	$1\frac{1}{2}$	
		$f(5) = 5k+1$ $5k+1 = 10 = 5k+1$	1	
		$5k+1 = 10$ $k = 9/5$	½	3
5.		$I = \int_0^3 x\,dx = \lim_{n\to\infty} \frac{3}{n}[0+(0+h)+(0+2h)+(0+3h)+\dots(0+(n-1)h)]$ $= \lim_{n\to\infty} \frac{3}{n}\cdot h[1+2+3+\dots n-1]$ $= \lim_{n\to\infty} \frac{3}{n}\cdot\frac{3}{n}\cdot\frac{n(n-1)}{2}$ $= \lim_{n\to\infty} \frac{9}{2}\left(\frac{n-1}{n}\right)$ $= \frac{9}{2}\lim_{n\to\infty}\left(1-\frac{1}{n}\right)$ $= \frac{9}{2}\cdot(1-0) = \frac{9}{2}$	3	3
6.		$\int_1^3 x^2\,dx = \left[\frac{x^3}{3}\right]_1^3 = \frac{1}{3}[3^3-1^3]$ $= \frac{1}{3}(27-1)$ $= \frac{26}{3}$	3	3
7.		$\vec{b_1} = i+2j+2k$ $\vec{b_2} = 3i+2j+6k$ $\vec{b_1}\times\vec{b_2} = \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 3 & 2 & 6 \end{vmatrix}$ $= i(12-4) - j(6-6) + k(2-6)$ $= 8i-4k$ $\lvert\vec{b_1}\times\vec{b_2}\rvert = \sqrt{8^2+4^2} = \sqrt{64+16} = \sqrt{80}$ $\frac{\vec{b_1}\times\vec{b_2}}{\lvert\vec{b_1}\times\vec{b_2}\rvert} = \frac{8i-4k}{\sqrt{80}}$ =	3	3

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8.	i)	c) $(1,1),(2,2),(3,3)$	1	
	ii)	$g(f(x)) = g(8x^3) = (8x^3)^{1/3} = 8^{1/3}(x^3)^{1/3} = 2x$	$1\frac{1}{2}$	
		$f(g(x)) = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$	$1\frac{1}{2}$	4
9.	i)	$\tan^{-1}(-1/\sqrt{3}) = -\pi/6$	1	
	ii)	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
	iii)	$\tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} = \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{13}}{1-\frac{1}{7}\times\frac{1}{13}}\right)$	1	
		$= \tan^{-1}\left(\frac{\frac{13+7}{7\times13}}{\frac{7\times13-1}{7\times13}}\right)$		
		$= \tan^{-1}\left(\frac{20}{91-1}\right)$		
		$= \tan^{-1}\left(\frac{2}{9}\right)$	1	4
10	i	$\lvert A\rvert = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$		
		$= 2(0-20)+3(-42-4)+5(30-0)$		
		$= -40-138+150 = -28$	2	
	ii)	$\lvert adj(A)\rvert = \lvert A\rvert^{3-1} = (-28)^2 = 784$	1	
	iii)	$\lvert 2A\rvert = 2^3\lvert A\rvert$		
		$= 8\times -28 = -224$	1	4
11.	i)	$y = \log x,\ x>0$		
		$\frac{dy}{dx} = \frac{1}{x}$	1	

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	ii)	$f(x)=2x^2-12x+1$ $f(2)=2\times 2^2-12\times 2+1=8-24+1=-15$ $f(4)=2\times 4^2-12\times 4+1=32-48+1=-15$ $f(2)=f(4)=-15$ $\exists$ a point $c\in(2,4)$ such that $f'(c)=0$ $f'(x)=4x-12$ $f'(c)=4c-12=0$ $4c=12$ $c=3\in(2,4)$ Rolle's theorem verified	1 1 1	4
12.		$\frac{2}{3}x^{2/3-1}+\frac{2}{3}y^{2/3-1}\cdot\frac{dy}{dx}=0$ $\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\cdot\frac{dy}{dx}=0$ $\frac{2}{3}y^{-1/3}\frac{dy}{dx}=-\frac{2}{3}x^{-1/3}$ $\frac{dy}{dx}=-\frac{x^{-1/3}}{y^{-1/3}}=-\frac{y^{1/3}}{x^{1/3}}$ $\left.\frac{dy}{dx}\right/_{(1,1)}=-\frac{1^{1/3}}{1^{1/3}}=-1$ Equation of the tangent $y-1=-1(x-1)$ $y-1+x-1=0$ $y+x=2$	1 1 1 1	4
13	i)	$\int\frac{2x}{1+x^2}\,dx$ $t=1+x^2$, $dt=2x\,dx$ $=\int\frac{dt}{t}=\log\lvert t\rvert+C$ $=\log\lvert 1+x^2\rvert+C$	1	1

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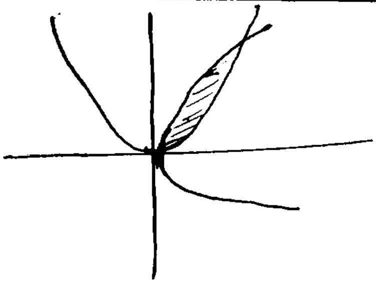
	ii)	$\int \frac{1}{\sqrt{x^2+2x-3}} = \int \frac{1}{\sqrt{x^2+2x+1-1-3}}$	1	
		$= \int \frac{1}{\sqrt{(x+1)^2-2^2}}$	1	
		$= \int \frac{1}{\sqrt{(x+1)^2-2^2}}$		
		$= \log\left\|(x+1)+\sqrt{x^2+2x-3}\right\|+c$	1	3
14.		 intersection points are (0,0) and (1,1)	1	
		Required area $= \int_0^1 (\sqrt{x}-x^2)dx$	1	
		$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1$	1	
		$= \left(\frac{2}{3} \cdot 1^{3/2} - \frac{1^3}{3}\right) - \left(\frac{0^{3/2}}{3/2} - \frac{0^3}{3}\right)$	1	
		$= \left(\frac{2}{3} - \frac{1}{3}\right) - 0$		
		$= \frac{1}{3}$		4
15.	i)	$\vec{a}\cdot\vec{b} = (5i-j-3k)\cdot(i+3j-5k)$	1	
		$= 5-3+15 = 20-3 = 17$	1	
	ii)	$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\|\vec{b}\|} = \frac{17}{\sqrt{5^2+(-1)^2+(-3)^2}\sqrt{1^2+3^2+(-5)^2}}$	1	
		$= \frac{17}{\sqrt{35}\sqrt{35}} = \frac{17}{35}$	1	
				4

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16		$\vec{a_2}-\vec{a_1} = (4i+5j+6k)-(i+2j+3k)$	1	
		$= 3i+3j+3k$		
		$\vec{b_1}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$		
		$= i(-3-6)-j(1-4)+k(3+6)$	1	
		$= -9i+3j+9k$		
		$\lvert\vec{b_1}\times\vec{b_2}\rvert = \sqrt{81+9+81} = \sqrt{171}$		
		Shortest distance $= \left\lvert\dfrac{(\vec{a_2}-\vec{a_1})\cdot\vec{b_1}\times\vec{b_2}}{\lvert\vec{b_1}\times\vec{b_2}\rvert}\right\rvert$	1	
		$= \left\lvert\dfrac{(3i+3j+3k)\cdot(9i+3j+9k)}{\sqrt{171}}\right\rvert$		
		$= \left\lvert\dfrac{27+9+27}{\sqrt{171}}\right\rvert$	1	
		$= \left\lvert\dfrac{73}{\sqrt{171}}\right\rvert$		4
17	i)	$\Sigma P(x) = 0+K+2K+2K+K = 1$	1	
		$6k = 1$		
		$k = 1/6$	1	
	ii)	$P(X<3) = P(X=0)+P(X=1)+P(X=2)$	1	
		$= 0+\frac{1}{6}+2\times\frac{1}{6}$		
		$= \frac{3}{6} = \frac{1}{2}$	1	4

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18	i)	$\begin{bmatrix} x+y & 2 \\ 5+x & 8 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 6 & 8 \end{bmatrix}$		
		$x+y=5$, $5+x=6$ $x=1$	1	
		$1+y=5$ $y=4$	1	
	ii)	$P=\frac{A+A^T}{2}=\frac{1}{2}\left[\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 4 \\ -1 & -6 & 3 \end{bmatrix}+\begin{bmatrix} 1 & 2 & -1 \\ 4 & 5 & -6 \\ -1 & 4 & 3 \end{bmatrix}\right]$	1	
		$=\frac{1}{2}\begin{bmatrix} 2 & 6 & -2 \\ 6 & 10 & -2 \\ -2 & -2 & 6 \end{bmatrix}$		
		$=\begin{bmatrix} 1 & 3 & -1 \\ 3 & 5 & -1 \\ -1 & -1 & 3 \end{bmatrix}$	1	
		$Q=\frac{A-A^T}{2}=\frac{1}{2}\left[\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 4 \\ -1 & -6 & 3 \end{bmatrix}-\begin{bmatrix} 1 & 2 & -1 \\ 4 & 5 & -6 \\ -1 & 4 & 3 \end{bmatrix}\right]$	1	
		$=\frac{1}{2}\begin{bmatrix} 0 & 2 & 0 \\ -2 & 0 & 10 \\ 0 & -10 & 0 \end{bmatrix}$		
		$=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 5 \\ 0 & -5 & 0 \end{bmatrix}$		
		$P+Q=\begin{bmatrix} 1 & 3 & -1 \\ 3 & 5 & -1 \\ -1 & -1 & 3 \end{bmatrix}+\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 5 \\ 0 & -5 & 0 \end{bmatrix}$	1	
		$=\begin{bmatrix} 1 & 4 & -1 \\ 2 & 5 & 4 \\ -1 & -6 & 3 \end{bmatrix}=A$		6

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19.		$AX = B$ $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	1	
		$X = A^{-1}B$ $\|A\| = 3(2-3) + 2(4+4) + 3(-6-4)$ $= -3 + 16 - 30 = -17 \neq 0$	1	
		A^{-1} exists. $\text{adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	1	
		$A^{-1} = \frac{1}{\|A\|} \text{adj } A.$ $= -\frac{1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	1	
		$X = A^{-1}B = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1	
		$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $x = 1,\ y = 2,\ z = 3$	1	6
20		$y = x^2 - 2x + 7$ $\frac{dy}{dx} = 2x - 2$	1	
		Parallel lines, slope $y = 2x + 9$ $m = 2$ $m = \frac{dy}{dx} \Rightarrow 2x - 2 = 2$ $2x = 4$ $x = 2$ $y = 2^2 - 2\times2 + 7 = 7$; $(2, 7)$	1	

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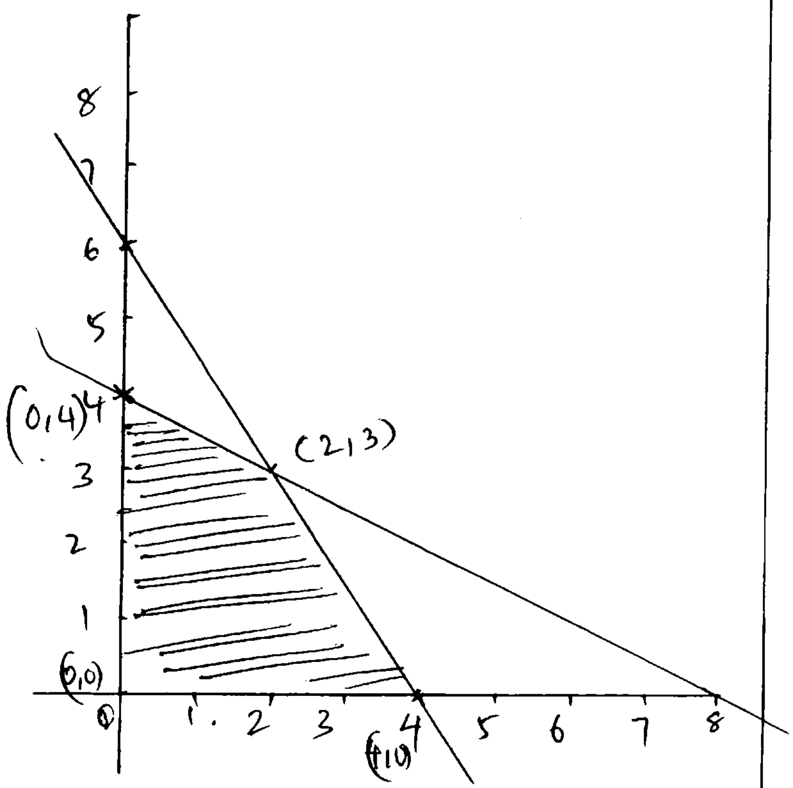
		Eqn of tangent line is $m=2, (x_1,y_1)=(2,7)$		
		$y-7=2(x-2)$		
		$y-7=2x-4$	1	
		$y-2x=3$		
	ii)	$P(x)=41-24x-6x^2$		
		$p'(x)=-24-12x$		
		$p'(x)=0 \Rightarrow -12x-24=0$		
		$-12x=24$	1	
		$x=-2$		
		$p''(x)=-12<0$		
		$\therefore x=-2$ is the point of maximum	1	
		Maximum value is		
		$P(-2)=41-24\times-2-6(-2)^2$		
		$=41+48-24$	1	6
		$=65$		
21	i)	order -2	1	
		degree -2	1	
	ii)	$\frac{dy}{dx}-\frac{y}{x}=\frac{2x^2}{x}$		
		$\frac{dy}{dx}-\frac{y}{x}=2x$		
		$P=-\frac{1}{x}, Q=2x$	1	
		$IF=e^{\int Pdx}=e^{\int -\frac{1}{x}dx}=e^{-\log x}=x^{-1}$		
		$=\frac{1}{x}$	1	
		$y.IF=\int Q.IF\,dx+C$	1	
		$y.\frac{1}{x}=\int 2x.\frac{1}{x}.dx+c$		
		$\frac{y}{x}=2\int 1\,dx+c$		
		$\frac{y}{x}=2x+c \Rightarrow y=2x^2+cx$	1	6

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22.	i	a) $\vec{AB} = (2-1)i + (1-2)j + (3-4)k$ $= i - 3j - k$	1	
		b) $\lvert\vec{AB}\rvert = \sqrt{1^2+(-3)^2+(-1)^2} = \sqrt{1+9+1} = \sqrt{11}$	1	
		unit vector of $\vec{AB} = \frac{i-3j-k}{\sqrt{11}}$	1	
	ii)	$\vec{AB} = -i - 4j + 4k \Rightarrow \lvert\vec{AB}\rvert = \sqrt{(-1)^2+(-4)^2+4^2}$ $= \sqrt{33}$	1	
		$\vec{BC} = 2i + 8j - 8k \Rightarrow \lvert\vec{BC}\rvert = \sqrt{2^2+8^2+(-8)^2}$ $= \sqrt{132} = 2\sqrt{33}$	1	
		$\vec{AC} = i + 4j - 4k \Rightarrow \lvert\vec{AC}\rvert = \sqrt{1^2+4^2+(-4)^2}$ $= \sqrt{33}$	1	
		$\lvert\vec{AB}\rvert + \lvert\vec{AC}\rvert = \sqrt{33} + \sqrt{33} = 2\sqrt{33} = \lvert\vec{BC}\rvert$ $\therefore$ A, B, C are collinear points.		6
23.		$x+2y=8$ $\begin{array}{\|c\|c\|c\|} \hline x & 0 & 8 \\ \hline y & 4 & 0 \\ \hline \end{array}$ $3x+2y=12$ $\begin{array}{\|c\|c\|c\|} \hline x & 0 & 4 \\ \hline y & 6 & 0 \\ \hline \end{array}$ 	4	

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		The corner points are $(4,0) \Rightarrow Z = -3\times4 + 4\times0 = -12$ $(0,4) \Rightarrow Z = -3\times0 + 4\times4 = 16$ $(2,3) \Rightarrow Z = -3\times2 + 4\times3 = 6$ $(0,0) \Rightarrow Z = 3\times0 + 4\times0 = 0$ The minimum point is $(4,0)$ and minimum value is -12	2	6
24.	i)	$P(E\cup F) = P(E) + P(F) - P(E\cap F)$ $0.68 = 0.6 + 0.2 - P(E\cap F)$ $P(E\cap F) = 0.8 - 0.68 = 0.12$ $P(E\cap F) = 0.12$ $P(E)\cdot P(F) = 0.6\times0.2 = 0.12$ $P(E\cap F) = P(E)\,P(F)$ E & F are independent.	1 1	2
	ii)	E_1 : Getting Box 1 E_2 : Getting Box 2 A : Getting a red ball $P(E_1) = \frac{1}{2}$ $\quad P(E_2) = \frac{1}{2}$ $P(A/E_1) = \frac{7}{10}$, $P(A/E_2) = \frac{4}{10}$ $P(E_1/A) = \frac{P(E_1)\,P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$ $= \frac{1/2 \times 7/10}{1/2\times 7/10 + 1/2\times 4/10} = \frac{7/20}{7/20 + 4/20} = \frac{7}{11}$	1 1 1 1	4