ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2023

PART-I/II/III

SUBJECT: MATHEMATICS

CODE NO: SY ~~254~~ 554 VERSION: S

80 SCORES 2½ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.	i)	Co. domain of f or Y	1	1
	ii)	$gof(x) = g(f(x))$	½	
		$= g(8x^3)$	½	
		$= [8x^3]^{1/3}$		
		$= 2x$		2.
		$fog(x) = f(g(x))$	½	
		$= f(x^{1/3})$	½.	
		$= 8[x^{1/3}]^3 = 8x$.		
2.	i)	3×3	1	1.
	ii)	$a_{ij} = 2i - j$		
		General Matrix $= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$	1	
		$\therefore \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 0 \end{bmatrix}$.	1	2.

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3.		$\begin{vmatrix} 3 & -1 & 2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix} = 3(0-5) - (-1)(0+3) + 2(0-0)$ $= 3(-5) + 1(3) + 2(0)$ $= -12.$	1 1 1	3.
4.		$\lim_{x \to 1^-} f(x) = 2 \times 1 + 3 = 5$ $\lim_{x \to 1^+} f(x) = 2 \times 1 + 3 = 5$ $f(1) = 2 \times 1 + 3 = 5$ $\therefore$ f is contenuous.	1 1 1	3.
5.	i.	$-e^{-x}$	1	1
	ii)	$2x + 3y = \sin x$ Diff w r t x $2 + 3\frac{dy}{dx} = \cos x$ $3\frac{dy}{dx} = \cos x - 2$ $\frac{dy}{dx} = \frac{\cos x - 2}{3}$.	1 1 1.	2

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6.		$f(x) = x^3 - 3x^2 + 4x$, $x \in R$ $f'(x) = 3x^2 - 6x + 4$ $= 3(x^2 - 2x + 1) + 1$ $= 3(x-1)^2 + 1 > 0$, in every interval of R . $\therefore$ the function f is increasing on R .	1 1 1	3.
7.	i)	A) 2	1	1
	ii)	$y = a\cos x + b\sin x$. $\rightarrow$ (1) diff. both sides of (1) w.r.t x . $\frac{dy}{dx} = -a\sin x + b\cos x$ $\frac{d^2y}{dx^2} = -a\cos x - b\sin x$. Substituting the values of $\frac{d^2y}{dx^2}$ and y in the given diff. equation we get LHS $= (-a\cos x - b\sin x) +$ $(a\cos x + b\sin x) = 0$ $=$ RHS .	$\frac{1}{2}$ $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$.	2 .

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8.	i)	$\frac{2}{\sqrt{29}}$	1	1
	ii)	Let $\vec{a}$ and $\vec{b}$ be the position vectors of the points A(−1, 0, 2) and B(3, 4, 6)		
		Then $\vec{a} = -i + 2k$	½	
		$\vec{b} = 3i + 4j + 6k$.	½	2.
		$\therefore \vec{b} - \vec{a} = 4i + 4j + 4k$.	½	
		let $\vec{r}$ be the position vector.		
		∴ then the vector equation		
		$\vec{r} = -i + 2k + \lambda(4i + 4j + 4k)$	½	
9.		$f(x) = 4x + 3$, $x \in R$		
		$f(x_1) = f(x_2)$		
		$4x_1 + 3 = 4x_2 + 3$.		
		$4x_1 = 4x_2$	1	
		$x_1 = x_2$.		
		$\Rightarrow$ f is one-one.		
		Let $y \in R$		4
		$y = 4x + 3$.	1	
		$x = \frac{y-3}{4} \in R$		
		f is onto.		
		So f is bijective and f is invertible.	1	
		$\therefore f^{-1} = \frac{y-3}{4}$	1	

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10.	i)	$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	1
	ii)	$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$	1	
		$2\tan^{-1}\frac{1}{2} = \tan^{-1}\left(\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right) = \tan^{-1}\frac{4}{3}$.		
		$\therefore \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{7}$		
		$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\cdot\frac{1}{7}}\right)$	1	3.
		$= \tan^{-1}\left(\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right)$	1	
		$= \tan^{-1}\left(\frac{31}{17}\right)$		
11.		$y = x^{\sin x}$		
		Taking log on both sides		
		$\log y = \log x^{\sin x}$	1	
		$\log y = \sin x \log x$.		4.
		diff w r t x.		
		$\frac{1}{y}\cdot\frac{dy}{dx} = \sin x\cdot\frac{1}{x} + \log x\cdot\cos x$	1	
		$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \log x\ \cos x\right]$	1	
		$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \log x\cos x\right]$	1	

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12.	i)	$\frac{1}{2}\sin 2x$.	1	1
	ii)	$\int \frac{x^3-1}{x^2}\,dx = \int x\,dx - \int x^{-2}\,dx$	1½	3
		$= \frac{x^{1+1}}{1+1} - \frac{x^{-2+1}}{-2+1} + c$	1	
		$= \frac{x^2}{2} + \frac{1}{x} + c$.	½.	
13.		Required area $= 2\int_0^{\pi/4} \cos 2x\,dx$	1½	4.
		$= 2\left[\frac{\sin 2x}{2}\right]_0^{\pi/4}$	1½	
		$= \frac{2}{2}\left[\sin \pi/2 - \sin 0\right]$	1	
		$= 1$		
		*. fig: 2 marks.		
14.	i)	$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$.		2
		Seperating the variables		
		$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$	1	
		Integrating both sides		
		$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$	1	
		$\tan^{-1} y = \tan^{-1} x + c$.		
		which is the general equation		

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	ii)	$\frac{dy}{dx} + Py = Q$ $P = \frac{2}{x}$, $Q = x$.	1	2.
		$IF = e^{\int \frac{2}{x} dx}$	1.	
		$= e^{2\log x}$. $= e^{\log x^2}$ $= x^2$.		
15.	i)	$\vec{a} = i + j - 2k$. $\vec{r} = xi + yj + zk$. If a, b, c are the direction ratio's of the vector $\vec{r} = xi + yj + zk$ then. $a = 1$, $b = 1$, $c = -2$.	1	2.
		$\therefore l = \frac{a}{\lvert\vec{r}\rvert} = \frac{1}{\sqrt{6}}$. $m = \frac{b}{\lvert\vec{r}\rvert} = \frac{1}{\sqrt{6}}$. $n = \frac{c}{\lvert\vec{r}\rvert} = \frac{-2}{\sqrt{6}}$. $\therefore$ direction cosines $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$	1.	
	ii)	$x = 2$ $y = 3$.	1 1	2.

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16.		Comparing the given equations of the plane with the equations $A_1x + B_1y + C_1z + D_1 = 0.$	1	
		$A_2x + B_2y + C_2z + D_2 = 0.$	1	
		$A_1 = 3 \quad B_1 = -6 \quad C_1 = 2$ $A_2 = 2 \quad B_2 = 2 \quad C_2 = -2$	½ ½.	4
		$\cos\theta = \left\| \frac{3\times2 + (-6)(2) + 2(-2)}{\sqrt{3^2+(-6)^2+(-2)^2}\,\sqrt{2^2+2^2+(-2)^2}} \right\|$	1	
		$= \left\| \frac{-10}{7\times2\sqrt{3}} \right\| = \frac{5}{7\sqrt{3}} = \frac{5\sqrt{3}}{21}.$		
		$\theta = \cos^{-1}\left(\frac{5\sqrt{3}}{21}\right).$		
17.		$SD = \frac{\left\| \begin{matrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{matrix} \right\|}{\sqrt{(a_2b_2-a_2b_1)^2 + (b_1c_2-b_2c_1)^2 + (a_1c_2-a_2c_1)^2}}$	2	
		$SD = \frac{\left\| \begin{matrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{matrix} \right\|}{\sqrt{(-8)^2+(-4)^2+(6)^2}}$	1	4
		$= \frac{116}{\sqrt{116}} = \sqrt{116}$ units.	1	

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18.	i)	$P(E) \times P(F) = 3/5 \times 1/3 = 1/5$ $= P(E \cap F)$ $\therefore$ E & F are independent	1.	1
	ii)	i) $16/81$	1	
		ii) $20/81$	1	3.
		iii) $40/81$.	1	
19.	i)	A	1	1
	ii)	$A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$	2	2
	iii	$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$	1	6.
		$5A = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$	1	3.
		$7I = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$	1	
		$A^2 - 5A + 7I = 0.$		

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20.		$A^{-1} = \frac{Adj.A}{\|A\|}$.	2	
		$Adj.A = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$	2	6.
		$\|A\| = -3$.	2	
21.	i)	$y = 3x^4 - 4x$ at $x = 4$	1	
		$\frac{dy}{dx} = 12x^3 - 4$.		
		at $x = 4$	1	
		$\frac{dy}{dx} = 12 \times 4^3 - 4$ $= 764$		
		$\therefore$ equation $y - y_0 = \frac{dy}{dx}(x - x_0)$.	1.	6.
	ii)	$\frac{dV}{dt} = 9\ cm^3/s$.	1	
		$\frac{dx}{dt} = \frac{3}{x^2}$	1	
		$\frac{ds}{dt} = \frac{36}{x}$.	1	
		when $x = 10$, $\frac{ds}{dt} = 3.6\ cm^2/s$.		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
22.	i.	$\int (4e^{3x} + 1)\, dx$.	2	6.
		$= \frac{4}{3} e^{3x} + x + C$.		
	ii)	$\int \frac{dx}{x^2 - 16} = \int \frac{dx}{x^2 - 4^2}$	2.	
		$= \frac{1}{8} \log \left\lvert \frac{x-4}{x+4} \right\rvert + c$.		
	iii)	$\int \frac{3x^2}{x^6+1} dx$. $= \tan^{-1} x^3 + C$.	2.	
		Put $x^2 = u$		
		$2x\, dx = du$.		
23.	i)	$\overrightarrow{AB} = j + k$, $\overrightarrow{AC} = i + 2j$		
		Area $= \frac{1}{2} \lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$		
		$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 1 & 2 & 0 \end{vmatrix}$	1	
		$= -4i + 2j - k$.		
		$\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert = \sqrt{21}$	1	
		$\therefore$ Area $= \frac{1}{2}\sqrt{21}$		

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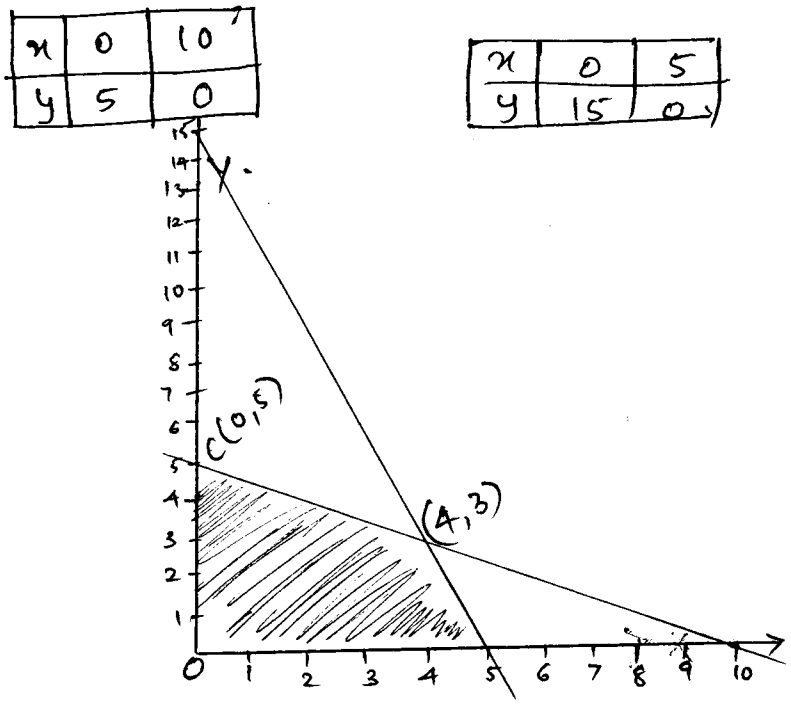
	ii)	$6\|\vec{a}\|^2 + 11\,\vec{a}\,\vec{b} - 35\|\vec{b}\|^2$	2.	
	iii)	$(\bar{a}+\bar{b}) \times (\bar{a}-\bar{b}) = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$	1	6.
		$= -2i + 4j - 2k.$		
		$\|\bar{c}\| = \sqrt{4+16+4}$		
		$= \sqrt{24}$	1/2	
		$= 2\sqrt{6}.$		
		$\frac{\bar{c}}{\|\bar{c}\|} = \frac{-1}{\sqrt{6}} i + \frac{2}{\sqrt{6}} j - \frac{1}{\sqrt{6}} k.$	1/2	
24.		$x + 2y = 10$ $3x + y = 15$	2	6.
			4	

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25.	i)	$4/11$	2	6.
	ii)	$4/5$	2	
	iii)	$2/3$.	2	