ANSWER KEY

Second YEAR HIGHER SECONDARY EXAMINATION March 2023

PART-I/II/III

SUBJECT: Mathematics.

CODE NO: SY ~~255~~ 555 **VERSION:** S

80 **SCORES** 2½ **HOURS**

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1		$f(x_1) = f(x_2)$	1	
		$4x_1 - 1 = 4x_2 - 1 \Rightarrow x_1 = x_2$	1	
		$y = 4x - 1$		
		$\Rightarrow \frac{y+1}{4}$	1	3
2.		$2X = \begin{bmatrix} 2 & 8 \\ 10 & 0 \end{bmatrix}$	1	
		$X = \begin{bmatrix} 1 & 4 \\ 5 & 0 \end{bmatrix}$	½	
		$2Y = \begin{bmatrix} 2 & 2 \\ 4 & 0 \end{bmatrix}$	1	
		$Y = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$	½	3
3.		$2 - 20 = 2x^2 - 24$	1	
		$-18 = 2x^2 - 24$	½	
		$2x^2 = 24 - 18$	½	
		$= 6$		3
		$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$	1	

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4		$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$	1	
		$\lim_{x\to 2} kx^2 = \lim_{x\to 2} 5$	1	
		$4k = 5$		
		$\Rightarrow k = 5/4$	1	3
5.	(i)	$\log x + c$	1	
	(ii)	$\int \frac{1-\sin x}{\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\,dx - \int \frac{\sin x}{\cos^2 x}\,dx$	1/2	
		$= \int \sec^2 x\,dx - \int \sec x \tan x\,dx$	1/2	
		$= \tan x - \sec x + c$	1	3
6.	(i)	(c) or 1	1	
	(ii)	$\bar{a}+\bar{b} = 5\hat{i} + \hat{j} - 4\hat{k}$	1	
		Unit vector $= \frac{5\hat{i}+\hat{j}-4\hat{k}}{\sqrt{25+1+16}}$	1	
		$= \frac{5}{\sqrt{42}}\hat{i} + \frac{1}{\sqrt{42}}\hat{j} - \frac{4}{\sqrt{42}}\hat{k}$		3
7.		Two lines are coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$	1	
		$\begin{vmatrix} -2 & -1 & 0 \\ -1 & 2 & 5 \\ -3 & 1 & 5 \end{vmatrix} = -2(10-5) + 1(-5+15) + 0$	1 + 1/2	
		$= -10 + 10 + 0$	1/2	
		$= 0$		3

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8.	(i)	(d) or reflexive, symmetric and transitive	1	
	(ii)	R is reflexive $\because (x,x) \in R \ \forall \ x \in A$ R is transitive, For $(x,y), (y,z) \in R \ (x,z) \in A$ R is not symmetric $\because (2,4) \in R$ but $(4,2) \notin R$	3	4
9.	(i)	$\frac{2\pi}{3}$	1	
	(ii)	$\tan^{-1} \frac{x+y}{1-xy}$	1	
	(iii)	$\tan^{-1} \frac{2}{11} + \tan^{-1} \frac{7}{24} = \tan^{-1} \left(\frac{\frac{2}{11} + \frac{7}{24}}{\left(1 - \frac{2}{11} \cdot \frac{7}{24}\right)} \right)$ $= \tan^{-1} \left(\frac{\frac{48+77}{264}}{\frac{264-14}{264}} \right)$ $= \tan^{-1} \left(\frac{125}{250} \right)$ $= \tan^{-1} \left(\frac{1}{2} \right)$	1 1	4
10.	(i)	$k^n \lvert A \rvert$	1	
	(ii)	$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $= \frac{1}{2} \begin{vmatrix} 2 & 4 & 1 \\ 3 & 3 & 1 \\ 1 & 5 & 1 \end{vmatrix} = \frac{1}{2} \cdot 0$ $\therefore \Delta = 0$ points are collinear	1 $\frac{1}{2} + \frac{1}{2}$ 1	4

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11.	(i)	$\frac{dy}{dx} = 2x \sec^2(x^2) \cdot \sec(\tan x^2) \tan(\tan x^2)$	2	
	(ii)	$\frac{dy}{d\theta} = -a\sin\theta$	½	
		$\frac{dx}{d\theta} = a(1-\cos\theta)$	½	
		$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{dx/d\theta} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-\sin\theta}{1-\cos\theta}$	1	
				4
12.		$f'(x) = 4x - 8$	1	
		$f'(x) = 0 \Rightarrow x = 2$	½	
		$x = 2$ divides $\mathbb{R}$ into $(-\infty, 2), (2, \infty)$	½	
		In $(-\infty, 2)$ $f'(x) < 0$ f is decreasing	1	
		In $(2, \infty)$ $f'(x) > 0$ f is increasing	1	4
13.		$I = \int_0^{\pi/2} \frac{\cos^4 x}{\sin^4 x + \cos^4 x} dx$	1	
		$I = \int_0^{\pi/2} \frac{\cos^4(\pi/2 - x)}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)} dx = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$	1	
		$2I = \int_0^{\pi/2} dx$	1	
		$\Rightarrow 2I = \pi/2 \Rightarrow I = \pi/4$	½ + ½	4

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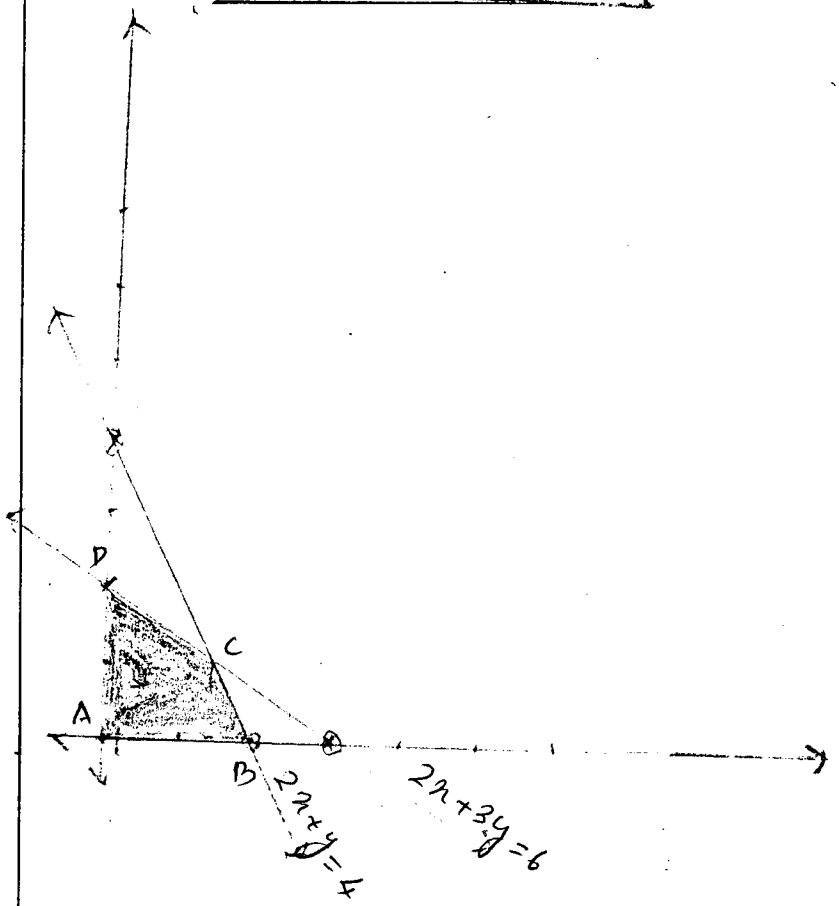
14.		Area of the circle $= 4 \times$ Area bounded by OABO $= 4 \int_0^2 y\,dx$ $= 4 \int_0^2 \sqrt{4-x^2}\,dx$ $= 4 \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2$ $= 4 \cdot 2\frac{\pi}{2}$ $= 4\pi$ sq. unit	1½ ½ ½ 1 ½	4
15.		$\vec{AB} = -\hat{i} - 2\hat{j} - 6\hat{k}$ $\vec{BC} = 2\hat{i} - \hat{j} + \hat{k}$ $\vec{CA} = -\hat{i} + 3\hat{j} + 5\hat{k}$ $\lvert\vec{AB}\rvert = \sqrt{41}$ $\lvert\vec{BC}\rvert = \sqrt{6}$ $\lvert\vec{CA}\rvert = \sqrt{35}$ $\lvert\vec{AB}\rvert^2 = 41 = 6 + 35 = \lvert\vec{BC}\rvert^2 + \lvert\vec{CA}\rvert^2$ $\therefore$ ΔABC is right angled triangle	1½ 1½ 1	4
16.		$d = \left\lvert \frac{(\bar{b}_1 \times \bar{b}_2)\cdot(\bar{a}_2 - \bar{a}_1)}{\lvert\bar{b}_1 \times \bar{b}_2\rvert} \right\rvert$ $\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 1 & -1 & 1 \end{vmatrix} = 3i - 3k$ $(\bar{a}_2 - \bar{a}_1) = -\hat{i} + 3\hat{j} + 2\hat{k}$ $\lvert\bar{b}_1 \times \bar{b}_2\rvert = \sqrt{18}$	1 1 ½ ½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$d = \left\| \frac{-3+0-6}{\sqrt{18}} \right\| = \frac{9}{\sqrt{18}}$	1	
		$= \frac{3}{\sqrt{2}}$ units		4
17		$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$		
		$A = \{2, 4, 6, 8, 10, 12\}$	1	
		$B = \{6, 7, 8, 9, 10, 11, 12\}$		
		$P(A\|B) = \frac{P(A \cap B)}{P(B)}$	1	
		$P(A \cap B) = 4/12 \quad P(B) = 7/12$	1	
		$\therefore P(A\|B) = \frac{4/12}{7/12}$		
		$= 4/7$	1	4
18.	(i)	$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix}$	2	
	(ii)	$A' = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ -1 & 1 & 3 \end{bmatrix}$	1	
		$A + A' = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$	1	
		$A - A' = \begin{bmatrix} 0 & -3 & -6 \\ 3 & 0 & -3 \\ 6 & 3 & 0 \end{bmatrix}$	1	
		$\frac{(A+A') + (A-A')}{2} = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{bmatrix} = A$	1	6

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
19.		$AX = B$	1	
		$\text{adj } A = \begin{bmatrix} 3 & 0 & -6 \\ 3 & 2 & -7 \\ 3 & 1 & -5 \end{bmatrix}$	2	
		$\lvert A \rvert = -3$	½	
		$A^{-1} = \frac{1}{\lvert A \rvert} \text{adj } A = \frac{1}{-3} \begin{bmatrix} 3 & 0 & -6 \\ 3 & 2 & -7 \\ 3 & 1 & -5 \end{bmatrix}$	1	
		$X = A^{-1}B = \frac{1}{-3} \begin{bmatrix} 3 & 0 & -6 \\ 3 & 2 & -7 \\ 3 & 1 & -5 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$	1	
		$= \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$	½	6
20	(i)	$\frac{dy}{dx} = e^x \cdot 2x + e^x(x^2-1)$	1	
		$= e^x \cdot 2x + y$	½	
		$\frac{d^2y}{dx^2} = \frac{dy}{dx} + 2 \cdot e^x (x+1)$	1	
		$= \frac{dy}{dx} + \frac{2y}{x^2-1}(x+1)$	1	
		$= \frac{dy}{dx} + \frac{2y}{x-1}$	½	4
	(ii)	$\frac{dy}{dx} = 5$	1	
		$2x - 1 = 5$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$x = 3$		
		$\Rightarrow y = 6$	1	2
		$(3,6)$ is required point.		(4+2=6)
21.	(i)	Order 2 degree 2	2	
	(ii)	$\frac{dy}{dx} - \frac{y}{x} = 2x$	1	
		$IF = e^{\int -\frac{1}{x} dx} = \frac{1}{x}$	1	
		Soln $y \cdot \frac{1}{x} = \int 2x \cdot \frac{1}{x} dx + C$	1	
		$\frac{y}{x} = 2x + C$	1	
		$y = 2x^2 + Cx$		(2+4=6)
22	(i)	Projection $= \frac{\bar{a} \cdot \bar{b}}{\lvert\bar{b}\rvert}$	1	
		$= \frac{1 \cdot 2 + 2 \cdot 3 + 1 \cdot 2}{\sqrt{4+9+4}} = \frac{2+6+2}{\sqrt{17}}$	1	
	(ii)	Area $= \frac{1}{2} \lvert \bar{a} \times \bar{b} \rvert$	1	
		$= \frac{1}{2} \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{vmatrix}$	1	
		$= \frac{1}{2} \lvert -\hat{i} - \hat{k} \rvert$	1	
		$= \frac{1}{2}\sqrt{2} = \frac{1}{\sqrt{2}}$ Sq. unit	1	2+4=6

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
23.		$2x + 3y = 6$ x: 0, 3 y: 2, 0 $2x + y = 4$ x: 0, 2 y: 4, 0  Corner Point / Value of $Z = 4x + 5y$ A (0, 0) — 0 B (2, 0) — 8 C (1.5, 1) — 11 D (0, 2) — 10 Max value of $Z = 11$ at (1.5, 1)	1 4 1	6

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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
24	(i)	$p + \frac{1}{2} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16} = 1$ $\Rightarrow p = 1 - \frac{8}{16} = 1/2$	1	
	(ii)	Mean, $E(x) = \sum x_i p_i$ $= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{8} + 4 \cdot \frac{1}{16} + 5 \cdot \frac{1}{16}$ $= \frac{31}{16}$	1 1	
	(iii)	Variance $= E(x^2) - (E(x))^2$ $E(x^2) = 1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{4} + 3^2 \cdot \frac{1}{8} + 4^2 \cdot \frac{1}{16} + 5^2 \cdot \frac{1}{16}$ $= \frac{83}{16}$ $\therefore E(x^2) - (E(x))^2 = \frac{83}{16} - \left(\frac{31}{16}\right)^2$ $= \frac{367}{256}$	1 1 1	(2+2+2) = 6