DITTO WS

ANSWER KEY

Code No	SY 551

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2023

PART-I/II/III

SUBJECT : MATHAMATICS (COMMERCE)

60 SCORES | 2 HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1		$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$	$\frac{1}{2}$	
		$a_{ij} = 2i + j$		
		$a_{11} = 3 \quad a_{12} = 4$		3
		$a_{21} = 5 \quad a_{22} = 6$	2	
		$\therefore A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$	$\frac{1}{2}$	
2	(i)	$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} \quad A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$		
		$A + A' = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$	$\frac{1}{2}$	
		$A - A' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$	$\frac{1}{2}$	
	(ii)	$P = \frac{1}{2}(A + A') = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$, is Symmetric	1	3
		$Q = \frac{1}{2}(A - A') = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, is Skew-Symmetric	1	
		$P + Q = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$		
		Remark : $P = \frac{1}{2}(A + A^T)$ $\frac{1}{2}$ score $Q = \frac{1}{2}(A - A^T)$ $\frac{1}{2}$ Score		

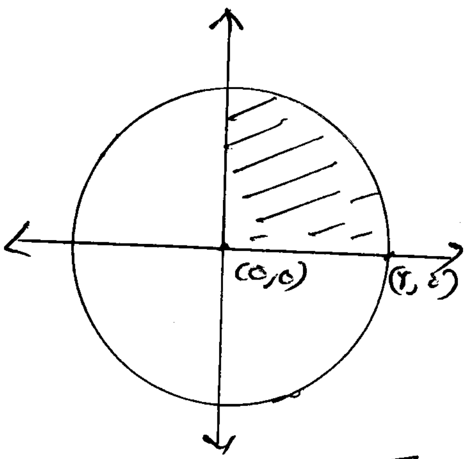
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
3	(i)	$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$		
		$2-20 = 2x^2-24$	$\frac{1}{2}$	3
		$x^2 = 3$	$\frac{1}{2}$	
		$x = \pm\sqrt{3}$		
	(ii)	Area of the $\Delta^{le} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$	1	
		$= \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$	$\frac{1}{2}$	
		$= \frac{47}{2}$ sq. units	$\frac{1}{2}$	
4	(i)	$\lim_{x\to 5^-} f(x) = \lim_{x\to 5} kx+1 = 5k+1$	1	3
		$\lim_{x\to 5^+} f(x) = \lim_{x\to 5} 3x-5 = 10$	1	
	(ii)	$\because$ f is Continuous		
		$\lim_{x\to 5^-} f(x) = \lim_{x\to 5^+} f(x)$	$\frac{1}{2}$	
		$5k+1 = 10$		
		$k = \frac{9}{5}$	$\frac{1}{2}$	

DITTO WS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
5		Let 'r' be the radius and 'A' be the area		3
		given $\frac{dr}{dt} = 3 cm/s$	$\frac{1}{2}$	
		$A = \pi r^2$	1	
		$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$	1	
		$= 6\pi r$		
		$\left(\frac{dA}{dt}\right)_{r=10} = 60\pi \ cm^2/sec$	$\frac{1}{2}$	
6	(i)	$\int \frac{1}{x} dx = \log\lvert x\rvert + c$	1	3
	(ii)	$\int \frac{x}{1+x^2} dx$		
		put $1+x^2 = u$	$\frac{1}{2}$	
		$2x\,dx = du$		
		$x\,dx = \frac{du}{2}$	$\frac{1}{2}$	
		$\int \frac{x}{1+x^2} dx = \frac{1}{2}\int \frac{1}{u} du$	$\frac{1}{2}$	
		$= \frac{1}{2}\log\lvert u\rvert + c$	$\frac{1}{2}$	
		$= \frac{1}{2}\log\lvert 1+x^2\rvert + c$		
7	(i)	$\vec{a}\cdot\vec{b} = 1\times 7 + 3\times -1 + 7\times 8$	$\frac{1}{2}$	3
		$= 60$	$\frac{1}{2}$	
		Remark : $\vec{a}\cdot\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$ give $\frac{1}{2}$ score		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
	(ii)	$\lvert\vec{b}\rvert = \sqrt{7^2+(-1)^2+8^2}$	$\frac{1}{2}$	
		$= \sqrt{114}$	$\frac{1}{2}$	
		Remark : $\lvert\vec{a}\rvert = \sqrt{x^2+y^2+z^2}$ give $\frac{1}{2}$ score		
	(iii)	Projection of $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$	$\frac{1}{2}$	
		$= \frac{60}{\sqrt{114}}$	$\frac{1}{2}$	
8	(i)	$P(A\cup B) = P(A)+P(B) - P(A\cap B)$	1	3
		$\frac{7}{11} = \frac{6}{11} + \frac{5}{11} - P(A\cap B)$	$\frac{1}{2}$	
		$P(A\cap B) = \frac{4}{11}$	$\frac{1}{2}$	
	(ii)	$P(A\mid B) = \frac{P(A\cap B)}{P(B)}$	$\frac{1}{2}$	
		$= \frac{4/11}{5/11} = \frac{4}{5}$	$\frac{1}{2}$	
9	(i)	$f(2) = 5$ $\quad f(-2) = 5$	1	
	(ii)	$f(2) = f(-2)$	$\frac{1}{2}$	
		$\therefore f$ is not 1-1	$\frac{1}{2}$	4
	(iii)	$-1 \in R$ in the Co-domain is not an image of any element of domain R	1	
		$\therefore f$ is not onto	1	
		Remark : for the concept of 'onto function' give 1 score		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
10		A B		4
	(i)	$\sin^{-1}\left(\frac{1}{2}\right)$ (c) $\frac{\pi}{6}$	1	
	(ii)	$\tan^{-1}(-1)$ (a) $-\frac{\pi}{4}$	1	
	(iii)	$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (e) $\frac{\pi}{4}$	1	
	(iv)	$\sec^{-1}(-2)$ (b) $\frac{2\pi}{3}$	1	
11	(i)	$A^2 = \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\4 & -2\end{bmatrix}$	$\frac{1}{2}$	4
		$= \begin{bmatrix}9-8 & -6+4\\12-8 & -8+4\end{bmatrix}$	1	
		$= \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix}$	$\frac{1}{2}$	
		Remark : $A^2 = A \cdot A$ give $\frac{1}{2}$ score		
	(ii)	$A^2 - A + 2I$		
		$= \begin{bmatrix}1 & -2\\4 & -4\end{bmatrix} - \begin{bmatrix}3 & -2\\4 & -2\end{bmatrix} + \begin{bmatrix}2 & 6\\0 & 2\end{bmatrix}$	1	
		$= \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} = 0$	1	
		Remark : For $I = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ give $\frac{1}{2}$ score		
12	(i)	$f'(x) = 2x+2$	1	4
	(ii)	$f'(x) = 0$	1	
		$2x+2 = 0$		
		$x = -1$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		In $(-\infty,-1)$, $f(x)$ is decreasing	$\frac{1}{2}$	
		and in $(-1,\infty)$, $f(x)$ is increasing	$\frac{1}{2}$	
		Remark: For definition of increasing & decreasing give 1 score		
13	(i)	$\int_0^1 \frac{1}{\sqrt{1-x^2}}dx = (\sin^{-1}x)_0^1$	1	4
		$= \sin^{-1}(1) - \sin^{-1}(0)$	$\frac{1}{2}$	
		$= \frac{\pi}{2} - 0 = \frac{\pi}{2}$	$\frac{1}{2}$	
	(ii)	$\int x \sin x\, dx = x\int \sin x\, dx - \int\left(\frac{d}{dx}(x)\int \sin x\, dx\right)dx$	1	
		$= -x\cos x - \int 1 \times -\cos x\, dx$	$\frac{1}{2}$	
		$= -x\cos x + \sin x + C$	$\frac{1}{2}$	
		Remark: For formula for integration by parts give 1 score		
14			$\frac{1}{2}$	
		$x^2+y^2=r^2 \Rightarrow y=\sqrt{r^2-x^2}$	1	
		Required Area $= 4\times$ Area of shaded region	$\frac{1}{2}$	
		$= 4\int_0^r y\, dx$		
		$= 4\int_0^r \sqrt{r^2-x^2}\, dx$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= 4\left[\frac{x}{2}\sqrt{r^2-x^2} + \frac{r^2}{2}\sin^{-1}\frac{x}{r}\right]_0^r$	$\frac{1}{2}$	
		$= 4 \cdot \frac{r^2}{2}\sin^{-1}(1) = 4 \cdot \frac{r^2}{2} \cdot \frac{\pi}{2}$	$\frac{1}{2}$	
		$= \pi r^2$		4
		Remark : for the formula $\int \sqrt{a^2-x^2}\,dx$ give 1 score Area $= \int_a^b y\,dx$ $\frac{1}{2}$ score for direct answer 'πr^2' give 1 score		
15	(i)	(c) 2	1	
	(ii)	$\frac{dy}{1+y^2} = (1+x^2)\,dx$	1	
		$\int \frac{dy}{1+y^2} = \int (1+x^2)\,dx$	1	4
		$\tan^{-1} y = x + \frac{x^3}{3} + c$	1	
16		$\vec{a_1} = \hat{i} + 2\hat{j} + 3\hat{k}$ $\quad \vec{b_1} = \hat{i} - 3\hat{j} + 2\hat{k}$ $\vec{a_2} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ $\quad \vec{b_2} = 2\hat{i} + 3\hat{j} + \hat{k}$	1	
		$\vec{a_2} - \vec{a_1} = 3\hat{i} + 3\hat{j} + 3\hat{k}$	$\frac{1}{2}$	
		$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$	$\frac{1}{2}$	
		$= -9\hat{i} + 3\hat{j} + 9\hat{k}$	$\frac{1}{2}$	
		$\lvert \vec{b_1} \times \vec{b_2} \rvert = \sqrt{171}$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$S.D = \left\lvert \frac{(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})}{\lvert \vec{b_1} \times \vec{b_2} \rvert} \right\rvert$	1	4
		$= \frac{9}{\sqrt{171}}$	$\frac{1}{2}$	
17	(i)	$AX = B$		6
		$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1	
	(ii)	$\text{Adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	2	
	(iii)	$\lvert A \rvert = 3 \times -1 + 2 \times 8 + 3 \times -10 = -17$	$\frac{1}{2}$	
		$A^{-1} = \frac{1}{\lvert A \rvert}(\text{adj} A)$	$\frac{1}{2}$	
		$= \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	$\frac{1}{2}$	
		$X = A^{-1} B$	$\frac{1}{2}$	
		$= \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	$\frac{1}{2}$	
		$= \frac{-1}{17} \begin{bmatrix} -8-5-4 \\ -64-6+36 \\ -80+1+28 \end{bmatrix}$		
		$= \frac{-1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	$\frac{1}{2}$	
		$x = 1, \quad y = 2, \quad z = 3$		
		Remark : In Qn (ii) for 7 correct entries give 2 scores		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
18	(i)	$2x + 3y = \sin x$		6
		D.w.r.t.x		
		$2 + 3\frac{dy}{dx} = \cos x$	1	
		$3\frac{dy}{dx} = \cos x - 2$	1	
		$\frac{dy}{dx} = \frac{\cos x - 2}{3}$		
	(ii)	$x = at^2 \qquad y = 2at$		
		$\frac{dx}{dt} = 2at \qquad \frac{dy}{dt} = 2a$	1	
		$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$	$\frac{1}{2}+\frac{1}{2}$	
	(iii)	$y = 2\sin x + 3\cos x$		
		$\frac{dy}{dx} = 2\cos x - 3\sin x$	1	
		$\frac{d^2y}{dx^2} = -2\sin x - 3\cos x$	$\frac{1}{2}$	
		$\frac{d^2y}{dx^2} = -y$	$\frac{1}{2}$	
		$\therefore \frac{d^2y}{dx^2} + y = 0$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
19				

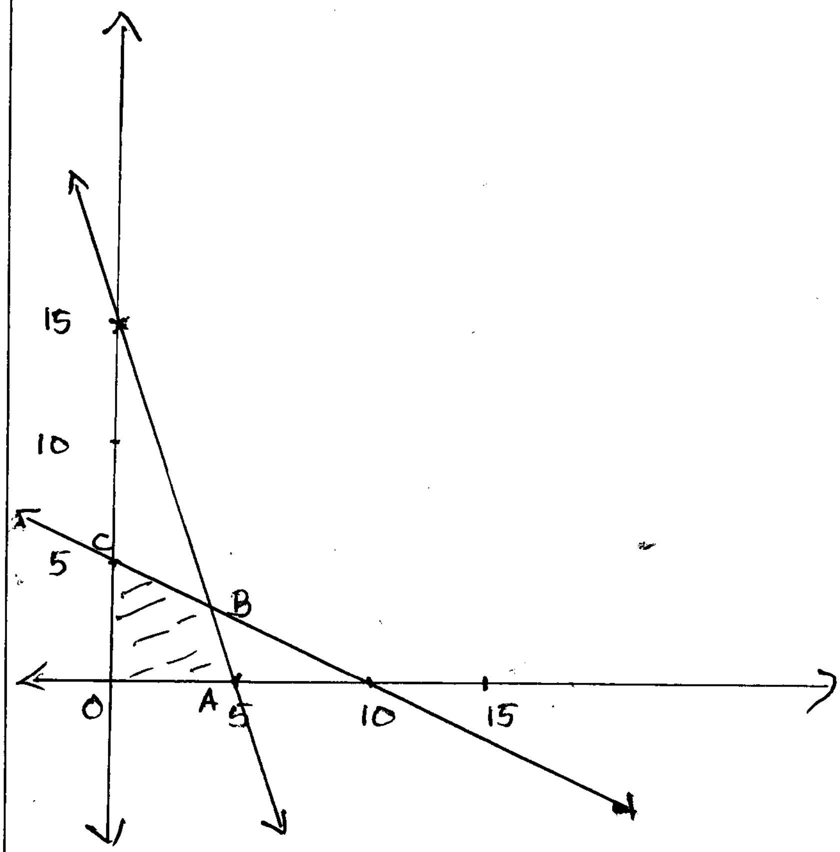
$x + 2y = 10$

x	0	10
y	5	0

$3x + y = 15$

x	0	5
y	15	0

Score: 1



Score: 4 — Total Score: 6

Corner points	$Z = 3x + 2y$
O (0, 0)	0
A (5, 0)	15
B (4, 3)	18
C (0, 5)	10

Maximum $Z = 18$ at $x = 4$, $y = 3$.

Score: 1

Remarks: 1) for x & y axis $\frac{1}{2}$ score each
2) for each correct line give $1\frac{1}{2}$ score
3) For incorrect shading and correct graph give $3\frac{1}{2}$ score

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
20	(i) a)	$P(A \text{ and } B) = P(A \cap B)$		
		$= P(A) \cdot P(B)$	$\frac{1}{2}$	
		$= 0.3 \times 0.6 = 0.18$	$\frac{1}{2}$	
	b)	$P(A \text{ and not } B) = P(A \cap B')$		
		$= P(A) \cdot P(B')$	$\frac{1}{2}$	
		$= 0.3 \times 0.4 = 0.12$	$\frac{1}{2}$	
		Remark: for $P(B')$ give $\frac{1}{2}$ score		
	ii)	E_1 : Selecting 1^{st} bag		
		E_2 : Selecting 2^{nd} bag		6
		$P(E_1) = P(E_2) = \frac{1}{2}$	$\frac{1}{2}$	
		A - Selecting Red ball	$\frac{1}{2}$	
		$P(A\|E_1) = \frac{3}{7}$ $\quad P(A\|E_2) = \frac{4}{9}$	1	
		Required Probability $= P(E_1\|A)$	$\frac{1}{2}$	
		$= \dfrac{P(E_1) \cdot P(A\|E_1)}{P(E_1) \cdot P(A\|E_1) + P(E_2) \cdot P(A\|E_2)}$	1	
		$= \dfrac{\frac{1}{2} \times \frac{3}{7}}{\frac{1}{2} \times \frac{3}{7} + \frac{1}{2} \times \frac{4}{9}} = \dfrac{27}{55}$	$\frac{1}{2}$	
		NB:- Give full score for alternative methods in any question		

1.	Leena. P. V 195871	9495216599
2.	J. Johnvictor (155126)	9446171748
3	Jiju Murali 155372	9809564586
4	V. R. Jayakumar (452374)	9446850319