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ANSWER KEY

Second YEAR HIGHER SECONDARY EXAMINATION 2023

PART-~~I/II~~/III

SUBJECT: Mathematics (HT)

CODE NO: SY ~~235~~ 575 VERSION: T

80 SCORES 2½ HOURS

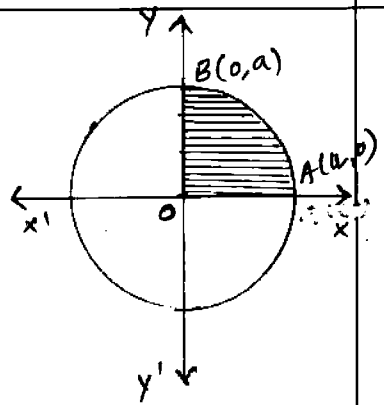
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1	i)	$f(x_1) = f(x_2)$		
		$\Rightarrow 2x_1 = 2x_2$	½	
		$\Rightarrow x_1 = x_2$	½	
	ii)	Let $y = f(x)$	½	3
		$\therefore y = 2x$		
		$x = y/2$	½	
		$\therefore f^{-1}(y) = y/2$	1	
		$\Rightarrow f^{-1}(x) = x/2$		
2		$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$		
		$a_{11} = 2\times1-1 = 1$		
		$a_{12} = 2\times1-2 = 0$	3	3
		$a_{21} = 2\times2-1 = 3$		
		$a_{22} = 2\times2-2 = 2$		
		$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
3		$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$		
		$3 - x^2 = 3 - 8$	1	3
		$\Rightarrow x^2 = 8$	1	
		$\Rightarrow x = \pm 2\sqrt{2}$	1	
4	i)	$x - y = \pi$		
		$1 - \frac{dy}{dx} = 0$		
		$\frac{dy}{dx} = 1$	1	
	ii)	$y = \tan(2x+3)$		3
		$\frac{dy}{dx} = \sec^2(2x+3) \cdot \frac{d}{dx}(2x+3)$	1	
		$= 2\sec^2(2x+3)$	1	
5		$y = A\sin x + B\cos x$		
		$\frac{dy}{dx} = A\cos x + B(-\sin x)$	1	
		$\frac{d^2y}{dx^2} = A(\sin x) + B(\cos x)$	1	3
		$= -[A\sin x + B\cos x]$		
		$= -y \Rightarrow \frac{d^2y}{dx^2} + y = 0$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
6	i).	C). 4	1	3
	ii)	$y-1 = m(x-1)$		
		$\Rightarrow y-1 = 4(x-1)$	1	
		$\Rightarrow y = 4x-3$	1	
7	i).	A). order = 2 degree = 1	1+1	3
	ii	B). 2	1	
8	i).	(1, -7, 4)	1	3
	ii).	$\vec{a} = 2\hat{i}+3\hat{j}+\hat{k}$	1	
		$\vec{b} = 3\hat{i}-4\hat{j}+5\hat{k}$		
		$\vec{b}-\vec{a} = \hat{i}-7\hat{j}+4\hat{k}$		
		required vector eqn is $\vec{r} = \vec{a}+\lambda(\vec{b}-\vec{a})$		
		$= 2\hat{i}+3\hat{j}+\hat{k}+\lambda(\hat{i}-7\hat{j}+4\hat{k})$	1	
9	i)	c). x^2+1	1	4
	ii	(1,1), (2,2), (3,3) lie in R	1	
		$\therefore$ R is reflexive		
		(1,2) $\in$ R but (2,1) $\notin$ R	1	
		$\therefore$ R is not symmetric		
		(1,2) $\in$ R & (2,3) $\in$ R but (1,3) $\notin$ R	1	
		$\therefore$ R is not transitive		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
10	i)	$\pi/4$	1	
	ii	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
		$\tan^{-1}\left(\frac{2}{11}\right)+\tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left[\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\times\frac{7}{24}}\right]$	1	4
		$= \tan^{-1}\left[\frac{125}{250}\right]$		
		$= \tan^{-1}(1/2)$	1	
11	i)	$\frac{dx}{d\theta} = a(1+\cos\theta)$	1	
		$\frac{dy}{d\theta} = a(0+\sin\theta) = a\sin\theta$		
		$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)}$	1	
		$= \frac{\sin\theta}{1+\cos\theta}$		4
	ii	$\log y = \sin x \log x$	1/2	
		$\frac{1}{y}\frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \log x \cdot \cos x$	1	
		$\frac{dy}{dx} = y\left(\frac{\sin x}{x} + \cos x \cdot \log x\right)$		
		$= x^{\sin x}\left(\frac{\sin x}{x} + \cos x \cdot \log x\right)$	1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
12		$I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\, dx \longrightarrow (1)$		
		Also $I = \int_0^{\pi/2} \frac{\sin^4(\pi/2 - x)}{\sin^4(\pi/2 - x) + \cos^4(\pi/2 - x)} \cdot dx$		
		$\therefore I = \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x + \sin^4 x}\, dx \longrightarrow (2)$	1	
		(1) + (2)		4
		$\Rightarrow 2I = \int_0^{\pi/2} 1 \cdot dx = [x]_0^{\pi/2}$	1	
		$= \pi/2 - 0$	1	
		$\therefore I = \pi/4$	1	
13		The whole area enclosed by the given circle = 4 x Area of the region AOBA		
		$= 4 \int_0^a y\, dx$	1	4
		$= 4 \int_0^a \sqrt{a^2 - x^2}\, dx$	1	
		$= 4 \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$	1	



Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= 4\left[\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}(1) - 0\right]$	½	
		$= 4 \times \frac{a^2}{2} \times \frac{\pi}{2}$		
		$= \pi a^2$	½	
14	i)	$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$		
		$\Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$	1	
		$\Rightarrow$ Integrating both sides		
		$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$	½	
		$\Rightarrow \tan^{-1} y = \tan^{-1} x + c$	½	4
	ii)	$P = 1/x \quad Q = x^2$	½	
		$IF = e^{\int P dx} = e^{\int 1/x dx} = e^{\log x} = x$	½	
		$y \times x = \int x^2 \times x \, dx + c$	½	
		$xy = \frac{x^4}{4} + c$	½	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
15		$\vec{a}+\vec{b}+\vec{c} = \vec{0}$		
		$\Rightarrow (\vec{a}+\vec{b}+\vec{c})^2 = 0$	1	
		$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$	1	
		$\Rightarrow 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}) = 0$	1	4
		$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -3/2$	1	
16		$\vec{a}_1 = \hat{i}+\hat{j} \quad \vec{b}_1 = 2\hat{i}-\hat{j}+\hat{k}$		
		$\vec{a}_2 = 2\hat{i}+\hat{j}-\hat{k} \quad \vec{b}_2 = 3\hat{i}-5\hat{j}+2\hat{k}$	1	
		$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$		
		$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$		
		$= 3\hat{i} - \hat{j} - 7\hat{k}$	1	4
		$\lvert \vec{b}_1 \times \vec{b}_2 \rvert = \sqrt{59}$		
		$S.D = \left\lvert \dfrac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{\lvert \vec{b}_1 \times \vec{b}_2 \rvert} \right\rvert$	1	
		$= \left\lvert \dfrac{3-0+7}{\sqrt{59}} \right\rvert = \dfrac{10}{\sqrt{59}}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
17		$A_1 = 3 \quad B_1 = -6 \quad C_1 = 2$ $A_2 = 2 \quad B_2 = 2 \quad C_2 = -2$	1	
		$\cos\theta = \left\lvert \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2} \cdot \sqrt{A_2^2+B_2^2+C_2^2}} \right\rvert$		
		$= \left\lvert \frac{3\times 2 + (-6)\times 2 + 2\times(-2)}{\sqrt{3^2+(-6)^2+2^2} \cdot \sqrt{2^2+2^2+(-2)^2}} \right\rvert$	1	4
		$= \left\lvert \frac{-10}{7\times 2\sqrt{3}} \right\rvert = \frac{5}{7\sqrt{3}}$		
		$= \frac{5\sqrt{3}}{21}$	1	
		$\theta = \cos^{-1}\left(\frac{5\sqrt{3}}{21}\right)$	1	
18	i)	$P(A/B) = \frac{P(A\cap B)}{P(B)}$	1/2	
		$= \frac{4/13}{9/13}$	1/2	
		$= \frac{4}{9}$	1	4
	ii	$P(A\cap B) = P(A)\times P(B)$	1	
		$= \frac{3}{5} \times \frac{1}{5}$	1/2	
		$= \frac{3}{25}$	1/2	

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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
19	i)	$A^2 = A \times A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$	1	
		$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$	½	
		$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	½	
		$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$	½	
		$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$	½	6
	ii)	$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$		
		$A^T = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$	½	
		$P = \frac{A + A^T}{2}$		
		$= \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$	1	
		$P^T = P$		
		$Q = \frac{A - A^T}{2} = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$		
		$= \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$	1	

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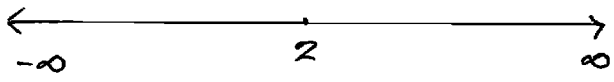
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$Q^T = -Q$ $P+Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ $= \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$	$\frac{1}{2}$	
20		The system of equations can be written in the form $Ax = B$ where $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$ $x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ $B = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ $\lvert A \rvert = -17 \neq 0$ $\text{adj } A = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$ $A^{-1} = \frac{1}{\lvert A \rvert} \text{adj } A$ $= \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$	1 1 $\frac{1}{2}$ 1	6

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$X = A^{-1}B$		
		$= \frac{-1}{17}\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}\begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$	1	
		$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{17}\begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$	1	
		$x = 1, \quad y = 2, \quad z = 3$	1/2	
21	i)	$f(x) = x^2 - 4x + 6$		
		$f'(x) = 2x - 4$	1	
		$f'(x) = 0 \Rightarrow x = 2$	1	
				
		$f'(x) = 2x - 4 > 0$ in the interval $(2, \infty)$	1	6
	ii)	$f'(x) = 12x^3 + 12x^2 - 24x$		
		$= 12x(x^2 + x - 2)$		
		$= 12x(x-1)(x+2)$	1/2	
		$f'(x) = 0$ at $x = 0, x = 1$ and $x = -2$		
		$f''(x) = 36x^2 + 24x - 24$	1/2	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= 12(3x^2+2x-2)$ $f''(0) = -24 < 0$ $f''(1) = 36 > 0$ $f''(-2) = 72 > 0$ By IInd derivative test $x=0$ is a Point of local maxima and local maximum value $= f(0) = 12$ while $x=1$ and $x=-2$ are the Points of local minima and local minimum Values of f at $x=1$ is $f(1)=7$ and at $x=-2$ is $f(-2) = -20$	1 1/2 1/2	
22	i).	$\frac{x^{n+1}}{n+1} + C$	1	6
	ii	$t = \tan^{-1}x \quad dt = \frac{dx}{1+x^2}$ $\therefore \int \frac{e^{\tan^{-1}x}}{1+x^2}dx = \int e^t dt$ $= e^t + C$ $= e^{\tan^{-1}x} + C$	1 1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
	iii)	$\int xe^x dx = xe^x - \int 1.e^x dx$	2	
		$= xe^x - e^x + c$	1	
23	i)	$[\vec{a}, \vec{b}, \vec{c}] = \begin{vmatrix} 1 & -2 & 3 \\ 2 & -3 & 1 \\ 3 & 1 & -2 \end{vmatrix}$	2	
		$= 1(6-1)+2(-4-3)+3(2+9)$	1/2	
		$= 5-14+33$		
		$= 24$	1/2	
	ii	$\hat{a} = \frac{\vec{a}}{\lvert\vec{a}\rvert}$	1/2	6
		$= \frac{2\hat{i}+3\hat{j}+\hat{k}}{\sqrt{2^2+3^2+1^2}}$		
		$= \frac{2\hat{i}+3\hat{j}+\hat{k}}{\sqrt{14}}$	1 1/2	
		$= \frac{2}{\sqrt{14}}\hat{i} + \frac{3}{\sqrt{14}}\hat{j} + \frac{1}{\sqrt{14}}\hat{k}$	1	

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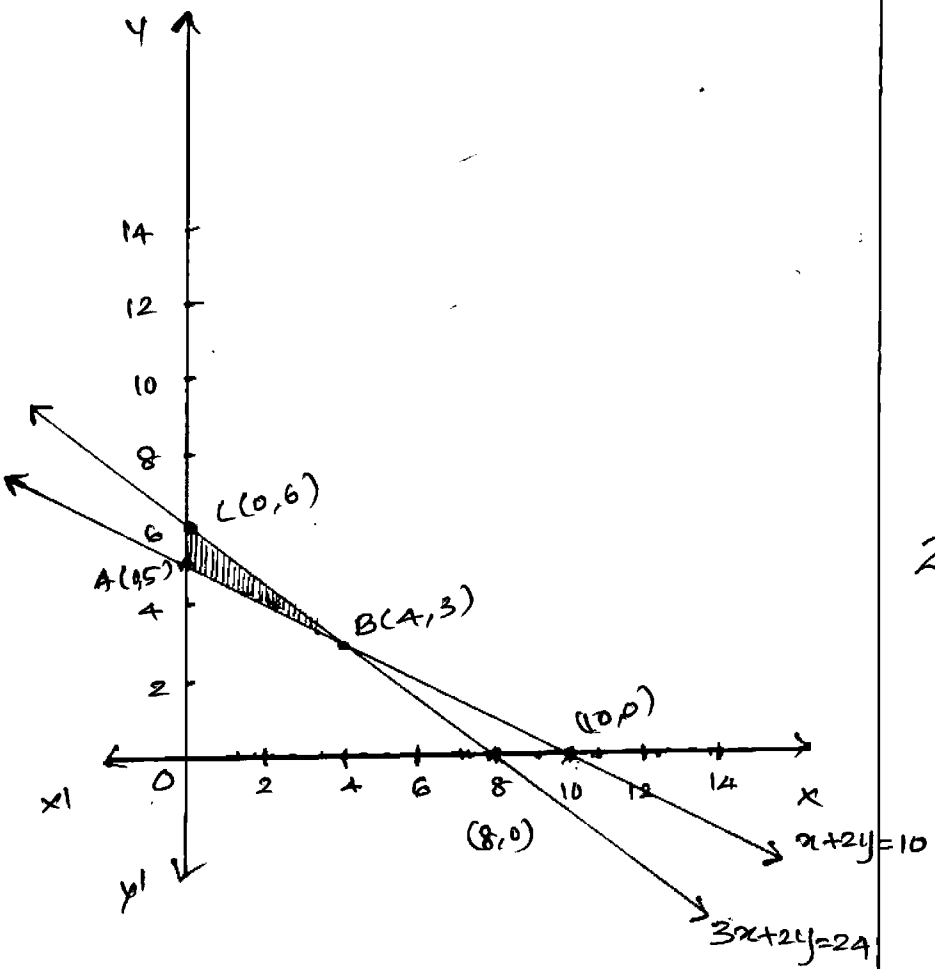
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
24		$x+2y=10$: x: 0, 10; y: 5, 0 — $3x+4y=24$: x: 0, 8; y: 6, 0	2	
			2	6
		Corner Point: (0,5), (4,3), (0,6) — Value of $Z=200x+500y$: 2500, 2300 ← Minimum, 3000	2	
		Hence minimum value of Z is 2300 at $x=4$ & $y=3$		

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Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
25	i)	$0.1+k+2k+2k+k=1$ $6k = 1-0.1 = 0.9$ $k = \frac{0.9}{6} = 0.15$	1	
	ii)	$\text{Mean} = E(x) = \sum x P(x)$ $= 0\times0.1 + 1k + 2\times2k + 3\times2k + 4\times k$ $= 0+k+4k+6k+4k$ $= 15k$ $= 15\times0.15$ $= 2.25$	2	6
	iii)	$E(x^2) = \sum x^2 P(x)$ $= 0^2\times0.1 + 1^2\times k + 2^2\times2k + 3^2\times2k + 4^2\times k$ $= 0+k+8k+18k+16k$ $= 43k$ $= 43\times0.15$ $= 6.45$	3	
		$\text{Var}(x) = E(x^2) - (E(x))^2$ $= 6.45 - (2.25)^2$ $= 1.39$		