Reg. No. : $\qquad$
Name : $\qquad$

## SECOND YEAR HIGHER SECONDARY EXAMINATION, MARCH 2022

## Part - III <br> Time : $21 / 2$ Hours <br> MATHEMATICS (SCIENCE) Cool-off time : 15 Minutes

Maximum : 80 Scores

## General Instructions to Candidates:

- There is a 'Cool-off time' of 15 minutes in addition to the writing time.
- Use the 'Cool-off time' to get familiar with questions and to plan your answers.
- Read questions carefully before answering.
- Read the instructions carefully.
- Calculations, figures and graphs should be shown in the answer sheet itself.
- Malayalam version of the questions is also provided.
- Give equations wherever necessary.
- Electronic devices except non-programmable calculators are not allowed in the Examination Hall.


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## PART-I

## A. Answer any 4 questions from 1 to 6. Each carries 1 score.

1. Let $R$ be the relation in the set $\{1,2,3,4\}$ given by $R=\{(1,1),(2,2),(3,3),(4,4)$, $(1,2),(1,3),(3,2)\}$
Choose the correct answer.
(a) R is reflexive and symmetric, but not transitive.
(b) R is reflexive and transitive, but not symmetric.
(c) R is symmetric and transitive, but not reflexive.
(d) R is an equivalence relation.
2. $\sin ^{-1} x+\cos ^{-1} x=$ $\qquad$ .
(a) 0
(b) $\frac{-\pi}{2}$
(c) $\frac{\pi}{2}$
(d) $\pi$
3. The slope of the tangent to the curve $\mathrm{y}=x^{2}+2$ at $x=2$ is $\qquad$ .
4. The order of the differential equation $\left(\frac{d y}{d x}\right)^{4}+3 y \frac{d^{2} y}{d x^{2}}=0$ is $\qquad$ -.
(a) 1
(b) 2
(c) 3
(d) 4
5. If $\vec{a}=\hat{i}+\hat{j}$ and $\vec{b}=3 \hat{j}+\hat{k}$, then $\vec{a} \cdot \vec{b}=$ $\qquad$ .
6. If vector equation of a line is $\vec{r}=(-3 \hat{i}+5 \hat{j}-6 \hat{k})+\lambda(2 \hat{i}+4 \hat{j}+2 \hat{k})$, then its Cartesian equation is $\qquad$ .
B. Answer all questions from 7 to 10. Each carries 1 score.
7. Principal value of $\tan ^{-1}(1)$ is $\qquad$ .
(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{2}$

## PART-I

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(c) R শ

2. $\sin ^{-1} x+\cos ^{-1} x=$ $\qquad$ .
(a) 0
(b) $\frac{-\pi}{2}$
(c) $\frac{\pi}{2}$
(d) $\pi$
 $\qquad$ (ேூ) 5 .
 $\qquad$ (ேロ)
(a) 1
(b) 2
(c) 3
(d) 4
5. $\vec{a}=\hat{i}+\hat{j}, \vec{b}=3 \hat{j}+\hat{k}$ ळ๑๐๐๐ $\vec{a} \cdot \vec{b}=$ $\qquad$ .

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(4 \times 1=4)
$$

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(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{2}$
8. Derivative of $\mathrm{e}^{2 x}$ w.r.t. $x$ is $\qquad$ .
9. For any two vectors $\vec{a}$ and $\vec{b},[\vec{a}, \vec{a}, \vec{b}]=$ $\qquad$ .
10. The direction ratios of the line passing through two points $(2,1,-2)$ and $(1,2,-3)$ are
$\qquad$ .

## PART-II

A. Answer any 3 questions from 11 to 15. Each carries 2 scores.
11. Find fog if $\mathrm{f}(x)=8 x^{3}$ and $\mathrm{g}(x)=x^{\frac{1}{3}}$, where f and g are real functions.
12. Find A if $2 \mathrm{~A}+\mathrm{B}=\left[\begin{array}{cc}4 & -2 \\ -1 & 3\end{array}\right]$ and $\mathrm{B}=\left[\begin{array}{ll}2 & 3 \\ 1 & 2\end{array}\right]$.
13. Show that the function $\mathrm{f}(x)=4 x+3$ is strictly increasing in $\mathbb{R}$.
14. Find a vector perpendicular to both $\vec{a}=5 \hat{i}-\hat{j}-3 \hat{k}$ and $\vec{b}=\hat{i}+3 \hat{j}-5 \hat{k}$.
15. Find the angle between the vectors $\vec{a}=\hat{i}-2 \hat{j}+3 \hat{k}$ and $\vec{b}=3 \hat{i}-2 \hat{j}+\hat{k}$.
B. Answer any 2 questions from 16 to 18. Each carries 2 scores.
16. Let ' $*$ ' be a binary operation on the set Q of rational numbers defined by $\mathrm{a} * \mathrm{~b}=\frac{\mathrm{ab}}{4}$. Check whether '*' is commutative or not.
17. Find the distance of the point $(2,3,1)$ from the plane $x+2 y+3 z=9$.
18. The random variable X has a probability distribution $\mathrm{P}(\mathrm{X})$ of the following form :

$$
\mathrm{P}(\mathrm{X})= \begin{cases}\mathrm{k}, & \text { if } \quad x=0 \\ 2 \mathrm{k}, & \text { if } \quad x=1 \\ 3 \mathrm{k}, & \text { if } x=2 \\ 0, & \text { otherwise }\end{cases}
$$

Determine the value of k .
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## PART－II

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$(2 \times 2=4)$






$$
\mathrm{P}(\mathrm{X})= \begin{cases}\mathrm{k}, & \text { if } \quad x=0 \\ 2 \mathrm{k}, & \text { if } \quad x=1 \\ 3 \mathrm{k}, & \text { if } \quad x=2 \\ 0, & \text { otherwise }\end{cases}
$$



## PART-III

A. Answer any 3 questions from 19 to 23. Each carries 4 scores. ( $3 \times 4=12$ )
19. Consider $\mathrm{f}: \mathbb{R} \rightarrow \mathbb{R}$ given by $\mathrm{f}(x)=2 x+3$. Show that f is invertible and find the inverse of $f$.
20. Find two positive numbers $x$ and $y$ such that their sum is 15 and sum of whose squares is minimum.
21. Find the area of the region bounded by $x^{2}=4 y, y=2, y=4$ and the $y$-axis in the first quadrant.
22. Find the general solution of the differential equation $x \frac{d y}{d x}+2 y=x^{2}, x \neq 0$
23. Find the shortest distance between the lines :

$$
\begin{aligned}
& \vec{r}=\hat{i}+\hat{j}+\lambda(2 \hat{i}-\hat{j}+\hat{k}) \text { and } \\
& \vec{r}=2 \hat{i}+\hat{j}-\hat{k}+\mu(3 \hat{i}-5 \hat{j}+2 \hat{k})
\end{aligned}
$$

B. Answer any 1 question from 24 to 25. Carries $\mathbf{4}$ scores.
24. Find the equation of the line joining the points $(1,2)$ and $(3,-1)$ using determinants.
25. Find the area between the curves $\mathrm{y}^{2}=x$ and $\mathrm{y}=x^{2}$.

## PART-IV

A. Answer any 3 questions from 26 to 29. Each carries 6 scores.
26. (i) Find the value of $\sin ^{-1} \sin \left(\frac{2 \pi}{3}\right)$
(ii) Prove that:

$$
\begin{equation*}
\tan ^{-1} \frac{2}{11}+\tan ^{-1} \frac{7}{24}=\tan ^{-1} \frac{1}{2} \tag{4}
\end{equation*}
$$

## PART－III

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23．$\vec{r}=\hat{i}+\hat{j}+\lambda(2 \hat{i}-\hat{j}+\hat{k})$




$(1 \times 4=4)$




## PART－IV

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（ii） $\tan ^{-1} \frac{2}{11}+\tan ^{-1} \frac{7}{24}=\tan ^{-1} \frac{1}{2}$ の $\operatorname{m\circ }$ คைைியிமலுக．
27. Find $\frac{d y}{d x}$
(i) $2 x+3 y=\sin y$
(ii) $x=\sin \mathrm{t}, \mathrm{y}=\cos 2 \mathrm{t}$
28. Integrate the following :
(i) $\frac{1}{x^{2}-6 x+13}$
(ii) $x \log x$
29. Solve the following Linear Programming Problem graphically :

Maximise $Z=3 x+2 y$
Subject to $x+2 \mathrm{y} \leq 10$

$$
\begin{aligned}
& 3 x+y \leq 15 \\
& x \geq 0, y \geq 0
\end{aligned}
$$

B. Answer any 2 questions from 30 to 32. Each carries 6 scores.
30. (i) Find $\frac{\mathrm{dy}}{\mathrm{d} x}$ if $\mathrm{y}=x^{\sin x}$
(ii) If $y=\left(\tan ^{-1} x\right)^{2}$, then show that $\left(1+x^{2}\right)^{2} y_{2}+2 x\left(1+x^{2}\right) y_{1}=2$
31. (i) Find $\int_{0}^{2} x^{2} \mathrm{~d} x$ as the limit of a sum.
(ii) Evaluate $\int_{0}^{\frac{\pi}{4}} \sin x \mathrm{~d} x$
32. Consider the differential equation $(x-y) \mathrm{dy}-(x+y) \mathrm{d} x=0$
(i) Show that it is homogeneous.
(ii) Solve this different equation.

(i) $2 x+3 y=\sin y$
(ii) $x=\sin \mathrm{t}, \mathrm{y}=\cos 2 \mathrm{t}$

(i) $\frac{1}{x^{2}-6 x+13}$
(ii) $x \log x$


Maximise $Z=3 x+2 y$
Subject to $x+2 \mathrm{y} \leq 10$

$$
3 x+y \leq 15
$$

$$
x \geq 0, y \geq 0
$$











## PART-V

Answer any 2 questions from 33 to 35. Each carries 8 scores.
33. Let $\mathrm{A}=\left[\begin{array}{ccc}2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0\end{array}\right]$
(i) Express A as the sum of a symmetric and a skew symmetric matrix.
(ii) Find $\mathrm{A}^{2}-5 \mathrm{~A}+6 \mathrm{I}$
34. Consider the matrix

$$
\mathrm{A}=\left[\begin{array}{ccc}
1 & 1 & 1 \\
0 & 1 & 3 \\
1 & -2 & 1
\end{array}\right]
$$

(i) Find Adj A
(ii) Prove that $\mathrm{A} \cdot \operatorname{Adj} \mathrm{A}=|\mathrm{A}| \mathrm{I}$
(iii) Solve the following system of equations using matrix method:

$$
\begin{align*}
x+y+z & =6 \\
y+3 z & =11 \\
x-2 y+z & =0 \tag{3}
\end{align*}
$$

35. (i) Given two independent events $A$ and $B$ such that $P(A)=0.3$ and $P(B)=0.6$. Find
(a) $\mathrm{P}(\mathrm{A}$ or B$)$
(b) P (neither A nor B$)$
(ii) A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball drawn is from the first bag.

## PART-V

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$(2 \times 8=16)$
33. $\mathrm{A}=\left[\begin{array}{ccc}2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0\end{array}\right]$ ®๑ற.









$$
\begin{gather*}
x+y+z=6 \\
y+3 z=11 \\
x-2 y+z=0 \tag{3}
\end{gather*}
$$


$\mathrm{P}(\mathrm{A})=0.3, \mathrm{P}(\mathrm{B})=0.6$ ชூ๑めコळ
(a) $\mathrm{P}(\mathrm{A}$ or B$)$
(b) P (neither A nor B$)$

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