ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2022

PART-I/II/III

SUBJECT: MATHEMATICS SCIENCE

CODE NO: ~~5X/S5S~~ 57-56 VERSION: R

80 SCORES 2½ HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.		b. R is reflexive and transitive but not symmetric	1	1
2.		c. $\frac{\pi}{2}$	1	1
3		4	1	1
4		b 2.	1	1
5		3	1	1
6.		$\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$	1	1
7.		b. $\frac{\pi}{4}$	1	1
8.		$2e^{2x}$	1	1
9		0	1	1
10		$-1, 1; 1$	1	1
11.		$f \cdot g(x) = f(g(x))$ $= f(x^{1/3})$ $= 8x$	1 ½ ½	2

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
12.		$2A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -5 \\ -2 & 1 \end{bmatrix}$	1	2
		$A = \begin{bmatrix} 1 & -5/2 \\ -1 & 1/2 \end{bmatrix}$	1	
13.		$f'(x) = 4$	1	2.
		$f'(x) > 0 \Rightarrow$ 'f' is strictly increasing	1	
14		$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & -3 \\ 1 & 3 & -5 \end{vmatrix}$	1	2
		$= 14\hat{i} + 22\hat{j} + 16\hat{k}$	1	
15		Let θ be the angle between $\vec{a}$ and $\vec{b}$		
		$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\lvert\vec{a}\rvert\lvert\vec{b}\rvert}$	1	2
		$= \frac{10}{\sqrt{14}\sqrt{14}} = \frac{10}{14} = \frac{5}{7}$	1	
16		$\vec{a} * \vec{b} = \frac{ab}{4}$		
		$\vec{b} * \vec{a} = \frac{ba}{4} = \frac{ab}{4}$	1	2.
		$\vec{a} * \vec{b} = \vec{b} * \vec{a}$.	1	
		$\therefore *$ is commutative.		
17.		Distance $= d = \left\lvert \frac{ax_1 + by_1 + cz_1 - d}{\sqrt{a^2+b^2+c^2}} \right\rvert$	1	2.
		$= \left\lvert \frac{2 + 2\times3 + 3\times1 - 9}{\sqrt{1+4+9}} \right\rvert = \frac{2}{\sqrt{14}}$	1	

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18		$\sum P(x_i) = 1$	1/2	
		$1k + 2k + 3k + 0 = 1$	1	
		$6k = 1$		2
		$k = 1/6$	1/2	
19		$y = 2x + 3 \Rightarrow x = \frac{y-3}{2}$		
		Define $g : \mathbb{R} \to \mathbb{R}$ by $g(y) = \frac{y-3}{2}$	1	
		$g \cdot f(x) = g(f(x)) = g(2x+3)$		
		$= \frac{2x+3-3}{2} = x$	1	
		$f \cdot g(x) = f(g(x)) = f\left(\frac{y-3}{2}\right)$		4
		$= 2 \cdot \frac{y-3}{2} + 3 = y$	1	
		$\therefore f$ is invertable and $g = f^{-1}$	1	
		[Give 1 score for $f^{-1}(x)$ directly]		
20.		$x + y = 15 \Rightarrow y = 15 - x$	1/2	
		$f(x) = x^2 + y^2 = x^2 + (15-x)^2$		
		$= 2x^2 - 30x + 225$	1	4
		$f'(x) = 4x - 30$	1/2	
		$f''(x) = 4$	1/2	
		$f'(x) = 0 \Rightarrow x = \frac{15}{2}$	1/2	
		$f''\left(\frac{15}{2}\right) = 4 > 0$	1/2	

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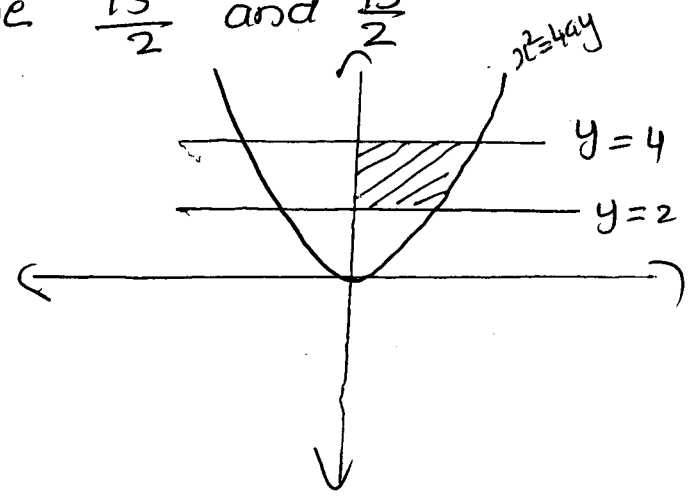
21		$\therefore f(x)$ is minimum at $x = \frac{15}{2}$		
		$x = \frac{15}{2} \Longrightarrow y = \frac{15}{2}$	½	
		The positive numbers required are $\frac{15}{2}$ and $\frac{15}{2}$		
			1	
		$\text{Area} = \int_2^4 x\,dy$		
		$= \int_2^4 2\sqrt{y}\,dy$	1	4
		$= 2 \cdot \frac{2}{3} y^{3/2} \Big]_2^4$	1	
		$= \frac{4}{3}\left[8 - \sqrt{8}\right]$	1	
		$= \frac{4}{3}\left[8 - 2\sqrt{2}\right]$ sq.unit		

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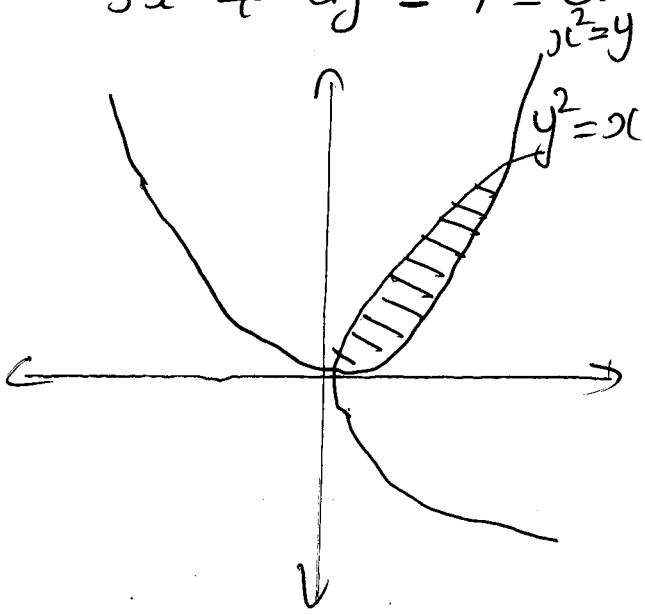
22.		$\frac{dy}{dx} + 2.\frac{y}{x} = x$ which is a linear differential equation where $P = \frac{2}{x}$ and $Q = x$.	1	
		$\int P\,dx = \int \frac{2}{x}\,dx = \log x^2$	1/2	
		$I.F = e^{\int P\,dx} = e^{\log x^2} = x^2$	1	
		The general solution is		4
		$y.I.F = \int Q.I.F\,dx$	1/2	
		$y\,x^2 = \int x^3\,dx$		
		$x^2 y = \frac{x^4}{4} + C.$	1	
23.		Comparing the given equations with $\vec{r} = \vec{a_1} + \lambda \vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu \vec{b_2}$		
		$\vec{a_1} = \hat{i} + \hat{j}$, $\vec{a_2} = 2\hat{i} + \hat{j} - \hat{k}$	1	
		$\vec{b_1} = 2\hat{i} - \hat{j} + \hat{k}$ $\vec{b_2} = 3\hat{i} - 5\hat{j} + 2\hat{k}$		
		$\vec{a_2} - \vec{a_1} = \hat{i} - \hat{k}$	1/2	
		$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$	1	4
		$\lvert \vec{b_1} \times \vec{b_2} \rvert = \sqrt{9+1+49} = \sqrt{59}$	1/2	
		$S.D = \frac{(\vec{a_2} - \vec{a_1}).(\vec{b_1} \times \vec{b_2})}{\lvert \vec{b_1} \times \vec{b_2} \rvert}$		
		$= \left\lvert \frac{3 - 0 + 7}{\sqrt{59}} \right\rvert = \frac{10}{\sqrt{59}}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
24.		Let (x, y) be a general point on the line. Then equation of the line is		
		$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & -1 & 1 \end{vmatrix} = 0$	2	
		$x(2-(-1)) - y(1-3) + 1(-1-6) = 0$	1	4
		$3x + 2y - 7 = 0.$	1	
25			1	
		The point of intersection is $(0,0)$ and $(1,1)$	1/2	4
		Required area $= \int_0^1 \sqrt{x}\,dx - \int_0^1 x^2 dx$	1	
		$= \frac{2}{3}x^{3/2}\Big]_0^1 - \frac{x^3}{3}\Big]_0^1$	1	
		$= \frac{2}{3} - \frac{1}{3}$	1/2	
		$= \frac{1}{3}$ sq.unit		

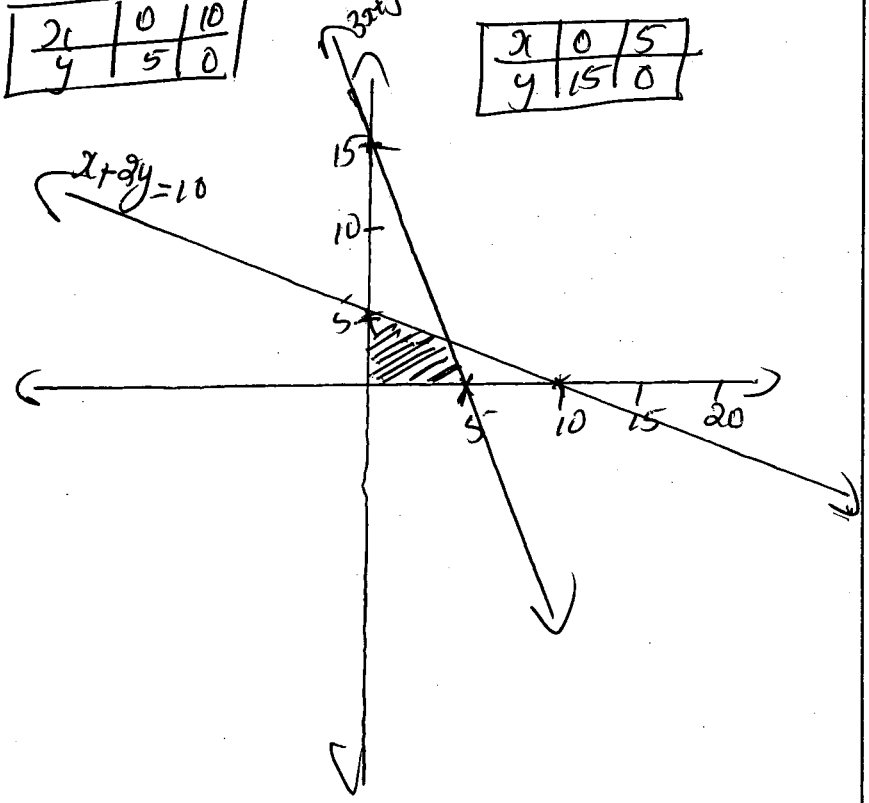
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
26	i	$\sin^{-1}\sin\frac{2\pi}{3} = \sin^{-1}\sin\left(\pi-\frac{\pi}{3}\right)$	1/2	
		$= \sin^{-1}\sin\frac{\pi}{3}$	1/2	
		$= \frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	1	
	ii)	$\tan^{-1}(x) + \tan^{-1}(y) = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
		$\tan^{-1}\left(\frac{2}{11}\right) + \tan^{-1}\left(\frac{7}{24}\right) = \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\frac{7}{24}}\right)$	1	6
		$= \tan^{-1}\left(\frac{125}{250}\right)$	1	
		$= \tan^{-1}\left(\frac{1}{2}\right)$	1	
27	i	$2x + 3y = \sin y$		
		$2 + 3\frac{dy}{dx} = \cos y \frac{dy}{dx}$	1	
		$2 = (\cos y - 3)\frac{dy}{dx}$	1	
		$\frac{dy}{dx} = \frac{2}{\cos y - 3}$	1	
	ii)	$x = \sin t \Rightarrow \frac{dx}{dt} = \cos t$	1	6
		$y = \cos 2t \Rightarrow \frac{dy}{dt} = -2\sin 2t$	1	
		$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin 2t}{\cos t}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
28	i	$\int \frac{dx}{x^2-6x+13} = \int \frac{dx}{(x-3)^2+2^2}$		
		$= \int \frac{dt}{t^2+2^2}$ $t = x-3$, $dt = dx$	1	
		We know $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$	1	
		$\int \frac{dx}{x^2-6x+13} = \frac{1}{2}\tan^{-1}\frac{t}{2}+C$	1	
		$= \frac{1}{2}\tan^{-1}\left(\frac{x-3}{2}\right)+C$		
	ii)	$\int x \log x\, dx = \int \log x \cdot x\, dx$		6
		$= \log x \int x\,dx - \int \frac{1}{x}\int x\,dx$	1	
		$= \log x \cdot \frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}dx$	1	
		$= \frac{x^2}{2}\log x - \frac{1}{2}\int x\,dx$		
		$= \frac{x^2}{2}\log x - \frac{x^2}{4}+C$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
29		$x + 2y = 10$ $3x + y = 15$		

x	0	10
y	5	0

x	0	5
y	15	0



1

3

Corner Pts	$Z = 3x + 2y$
O (0,0)	0
A (5,0)	15
B (4,3)	18
C (0,5)	10

2

6

Maximum value = 18
at (4,3)

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30	i	$y = x^{\sin x}$		
		$\log y = \log x^{\sin x}$		
		$= \sin x \log x$	1	
		$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{1}{x} + \log x \cdot \cos x$	1	
		$\frac{dy}{dx} = y\left[\frac{\sin x}{x} + \log x \cdot \cos x\right]$	1	
		$= x^{\sin x}\left[\frac{\sin x}{x} + \log x \cos x\right]$		
	ii)	$y = (\tan^{-1} x)^2$		6
		diff w.r.to x		
		$y_1 = 2\tan^{-1} x \frac{1}{1+x^2}$.	1	
		$(1+x^2)y_1 = 2\tan^{-1} x$	1/2	
		$(1+x^2)y_2 + 2x y_1 = \frac{2}{1+x^2}$	1	
		$(1+x^2)^2 y_2 + 2x(1+x^2)y_1 = 2$.	1/2	
31.	i)	$\int_a^b f(x)\,dx = (b-a)\lim_{n\to\infty}\frac{1}{n}[f(a)+$	1	
		$f(a+h)+\cdots+f(a+n-1h)]$		
		where $h = \frac{b-a}{n}$		
		Here $a=0$, $b=2$, $f(x)=x^2$, $h=\frac{2}{n}$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\int_0^2 x^2 dx = 2 \lim_{n\to\infty} \frac{1}{n} \left[f(0) + f\left(\frac{2}{n}\right) + \cdots + f\left(\frac{2(n-1)}{n}\right) \right]$	1	
		$= 2 \lim_{n\to\infty} \frac{1}{n} \left[0 + \frac{2^2}{n^2} + \frac{4^2}{n^2} + \cdots + \frac{(2n-2)^2}{n^2} \right]$		
		$= 2 \lim_{n\to\infty} \frac{1}{n} \left[\frac{2^2}{n^2} \left\{ 1 + 2^2 + \cdots + (n-1)^2 \right\} \right]$		
		$= 2 \lim_{n\to\infty} \frac{1}{n} \left[\frac{4}{n^2} \times \frac{n(n-1)(2n-1)}{6} \right]$	1	6
		$= 2 \lim_{n\to\infty} \frac{4}{n^2} \times \frac{n^2\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)}{6}$		
		$= \frac{8}{3}$	1	
	ii)	$\int_0^{\pi/4} \sin x \, dx = -\cos x \Big]_0^{\pi/4}$	1	
		$= -\cos\frac{\pi}{4} + \cos 0$	1/2	
		$= 1 - \frac{1}{\sqrt{2}}$	1/2	
32.		$\frac{dy}{dx} = \frac{x+y}{x-y}$		
		Put $y = vx$, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$	1	
		$v + x\frac{dv}{dx} = \frac{x + vx}{x - vx}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$v + x\frac{dv}{dx} = \frac{1+v}{1-v}$	1	
		$x\frac{dv}{dx} = \frac{1+v^2}{1-v}$	1	
		$\frac{1-v}{1+v^2}dv = \frac{dx}{x}$		
		$\int \frac{1}{1+v^2} - \int \frac{v}{1+v^2}dv = \int \frac{dx}{x}$	1	6
		$\tan^{-1}(v) - \frac{1}{2}\log\lvert 1+v^2\rvert = \log\lvert x\rvert + c.$		
		$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left\lvert 1+\frac{y^2}{x^2}\right\rvert = \log\lvert x\rvert + c.$	1	
33	i)	$A' = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$	1	
		$P = \frac{A+A'}{2} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$	1	
		$P' = P \Longrightarrow P$ is symmetric	1/2	
		$Q = \frac{A-A^T}{2} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$	1	
		$Q' = -Q \Longrightarrow Q$ is skew symmetric	1/2	
		$P+Q = A$		
	ii)	$A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$	1	

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		$5A = \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$, $6I = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$	2	8
		$A^2 - 5A + 6I = \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$	1	
34)	i.	$Adj\ A = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$	2	
	ii	$A \cdot Adj\ A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$	1	
		$\lvert A \rvert = 9$	1	
		$\lvert A \rvert I = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$	1	
	iii)	The system of equation can be represented as $AX = B$		8
		where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$	1	
		$X = A^{-1}B$		
		$A^{-1} = \frac{adj\ A}{\lvert A \rvert} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$X = A^{-1}B = \frac{1}{9}\begin{bmatrix}9\\18\\27\end{bmatrix} = \begin{bmatrix}1\\2\\3\end{bmatrix}$	1	
		$x = 1,\ y = 2,\ z = 3$		
35	I	i) $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$	1	
		$= 0.3 + 0.6 - 0.18$	1	
		$= 0.72$.	1	
		ii) $P(A' \cap B') = 1 - P(A \cup B)$	1	
		$= 0.28$	1	
	II	Let E_1 and E_2 be the event of choosing bag I and II respectively. A be the event drawing a red ball.		8
		$P(E_1) = P(E_2) = \frac{1}{2}$	1	
		$P(A \mid E_1) = \frac{4}{8} = \frac{1}{2}$	1	
		$P(A \mid E_2) = \frac{2}{8} = \frac{1}{4}$	1	
		Using Baye's theorem.		
		$P(E_1 \mid A) = \frac{P(E_1)P(A \mid E_1)}{P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)}$	1	
		$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{8}} = \frac{4}{5}$		

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		Prepared by Deepa mol. V.C - 9496745509 Deepa.		