ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION March 2022

PART-I/II/III

SUBJECT: Mathematics - Science (HI)

CODE NO: 8465 VERSION: R

60 SCORES 2 HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1		$f^{-1} = \{(2,1)(1,2)(1,3)\}$		1
2.		(ii) $\frac{\pi}{4}$.		1
3		$\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} = 2\times2 - 4\times-1 = 8$		1
4		(iii) $\tan^{-1}x$.		1
5		(ii) $\int_a^b y\,dx$		1
6		2		1
7		$\lvert\vec{a}\rvert = \sqrt{1^2+1^2+1^2} = \sqrt{3}$		1
8		(ii) (0,1,0)		1
9		(iii) not defined.		1
10		2 x 3		1
11		(ii) $3^2\lvert A\rvert$		1
12		(iv) $2\cos 2x$.		1
13		(i) $e-1$		1

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14		$x+3=2 \quad y-7=6$	1	2.
		$x=-1 \quad y=13$	1	
15.		$2A+B = 2\begin{bmatrix}1 & 3\\ 2 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\ -1 & 0\end{bmatrix}$		
		$= \begin{bmatrix}2 & 6\\ 4 & 10\end{bmatrix}+\begin{bmatrix}3 & 0\\ -1 & 0\end{bmatrix}$	1	2
		$= \begin{bmatrix}5 & 6\\ 3 & 10\end{bmatrix}$	1	
16.		Left limit $= \lim_{x\to 1^-} x^9+1 = 1+1=2$	1	2
		Right limit $= \lim_{x\to 1^+} x+1 = 1+1=2$	1	
		Yes. $f(x)$ is continuous at $x=1$		
17		Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\lvert\vec{b}\rvert}$	1	2
		$= \frac{1\times 7+2\times -1+3\times 8}{\sqrt{49+1+64}}$	1	
		$= \frac{29}{\sqrt{114}}$		

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18		$2x + 3y = \sin x$		
		$2 \times 1 + 3\frac{dy}{dx} = \cos x$	1	2
		$\frac{dy}{dx} = \frac{\cos x - 2}{3}$	1	
19		$y = e^x + 1$		
		$\frac{dy}{dx} = e^x$, $\frac{d^2y}{dx^2} = e^x$	1	2
		$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x - e^x = 0$	1	
20		$\vec{r} = \vec{a} + \lambda \vec{b}$	1	
		$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$	1	2
21		$a * b = \frac{ab}{4}$		
		$2 * 3 = \frac{2 \times 3}{4} = \frac{6}{4}$	1	
		$a * b = \frac{ab}{4}$		3
		$= \frac{ba}{4}$	2	
		$= b * a$ Yes commutative		

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22.		$a_{ij} = \hat{i} + 3\hat{j}$		
		$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$	1	3
		$A = \begin{bmatrix} 4 & 7 \\ 5 & 10 \end{bmatrix}$	2	
23		$\frac{dy}{dx} = \frac{x+1}{2+y}$		
		$(2+y)dy = (x+1)\,dx$	1	
		Solution is $\int (2+y)dy = \int (x+1)dx$	1	3
		$2y + \frac{y^2}{2} = \frac{x^2}{2} + x + c$	1	
24		unit vector along $\vec{a} = \frac{\vec{a}}{\lvert\vec{a}\rvert}$	1	
		$= \frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}$	1	3
		vector of magnitude 7 $= 7\left(\frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}\right)$	1	

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25.		$y = 4\sin x + 3\cos x$		
		$\frac{dy}{dx} = 4\cos x - 3\sin x$	1	
		$\frac{d^2y}{dx^2} = -4\sin x - 3\cos x$	1	3
		$\frac{d^2y}{dx^2} + y = -4\sin x - 3\cos x + 4\sin x + 3\cos x$ $= 0$	1	
26		$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\lvert\vec{a}\rvert \lvert\vec{b}\rvert}$	1/2	
		$\vec{a} \cdot \vec{b} = 1\times1 + 1\times-1 + -1\times1$ $= -1$	1	3
		$\lvert\vec{a}\rvert = \sqrt{3} \quad \lvert\vec{b}\rvert = \sqrt{3}$	1	
		$\theta = \cos^{-1}\left(\frac{-1}{3}\right)$	1/2	
27.	i)	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $= \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11}$	1	
	ii)	$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{4}{11}}{\frac{5}{11}} = \frac{4}{5}$	2	

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28.	i)	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	
	ii)	$LHS = \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11}$		
		$= \tan^{-1}\left(\frac{\frac{1}{2}+\frac{2}{11}}{1-\frac{1}{2}\times\frac{2}{11}}\right)$	1	
		$= \tan^{-1}\left(\frac{\frac{15}{22}}{\frac{20}{22}}\right)$	1	4
		$= \tan^{-1}\frac{3}{4} = RHS.$	1	
29.		$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$	1	
		$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$	1	4
		$= \frac{1}{2}(3(2-1) - 8(-4-5) + 1(-4-10)$		
		$= \frac{1}{2}(3\times1 - 8\times-9 + 1\times-14)$		
		$= \frac{1}{2}(3+72-14)$	2	
		$= \frac{1}{2}\times 61$		

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30		$f(x) = x^2 - 4x + 6$		
		$f'(x) = 2x - 4$		
		$f'(x) = 0 \Rightarrow 2x = 4$ $x = 2$.		
		intervals $(-\infty, 2)$ and $(2, \infty)$	2	4
	i)	$f(x)$ is increasing in $(2, \infty)$	1	
	ii)	$f(x)$ is decreasing in $(-\infty, 2)$	1	
31		$I = \int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x}\, dx$ — ①		
		Also $I = \int_0^{\pi/2} \frac{\sin^5(\pi/2 - x)}{\sin^5(\frac{\pi}{2} - x) + \cos^5(\frac{\pi}{2} - x)}\, dx$		
		$= \int_0^{\pi/2} \frac{\cos^5 x}{\cos^5 x + \sin^5 x}\, dx$ — ②	1	
		① + ② $\Rightarrow 2I = \int_0^{\pi/2} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x}\, dx$	1	4
		$= \int_0^{\pi/2} 1\, dx$		
		$I = \pi/4$	1	

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32.		$AX = B$		
		$\begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$	½	
		$\lvert A \rvert = \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} = 4 - 15 = -11$	½	
		Cofactor matrix $= \begin{bmatrix} 2 & -3 \\ -5 & 2 \end{bmatrix}$		4
		$\text{adj } A = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$		
		$A^{-1} = \dfrac{\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}}{-11}$	2	
		$X = A^{-1} \times B = \dfrac{-1}{11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix}$		
		$= \begin{bmatrix} \frac{2-35}{-11} \\ \frac{-3+14}{-11} \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$	1	
33.		$A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$	½	
	i) ii)	$A + A' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$	½	4
		$(A+A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$ $\therefore$ symmetric	1	

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33	ii)	$A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$	1	
		$(A - A')' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ skew symmetric	1	
34.		$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$		
		$= \begin{bmatrix} 9+-1 & 3+2 \\ -3+-2 & -1+4 \end{bmatrix}$		
		$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$	3	6
		$A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$		
		$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	3	

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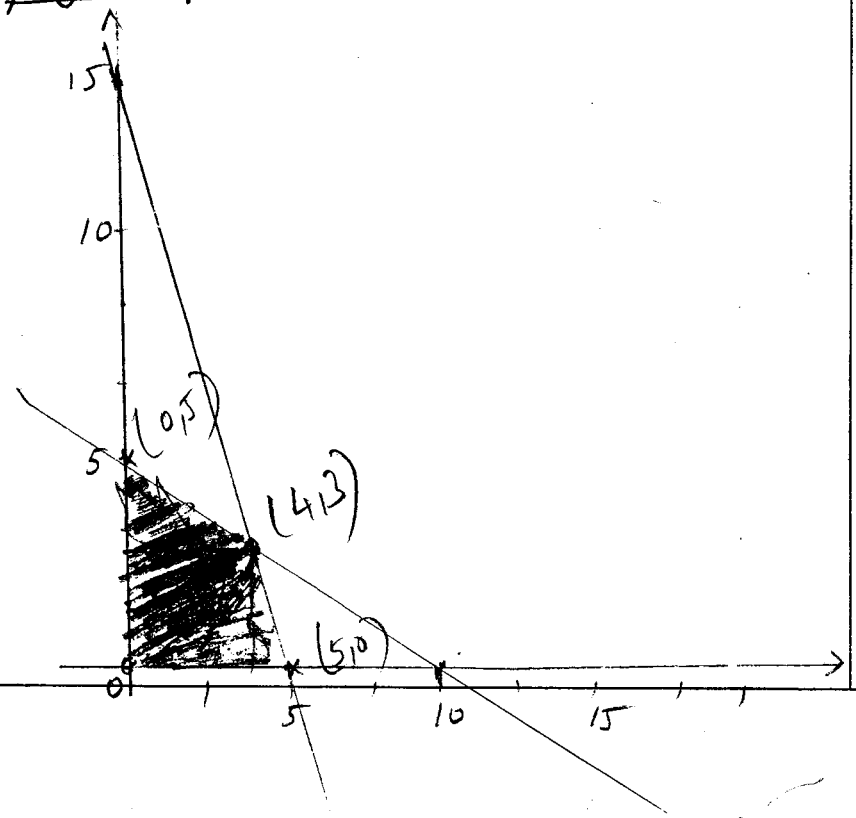
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
35.	i)	Cofactor matrix $= \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$	1½	
		$adjA = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$	1½	6
	ii)	$\lvert A \rvert = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$	1½	
		$A^{-1} = \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2}$	1½	
36		$x + 2y = 10$ | x | 0 | 10 | | y | 5 | 0 | $3x + y = 15$ | x | 0 | 5 | | y | 15 | 0 | 	4.	6

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		at (5,0) $z = 3\times5 + 2\times0 = 15$ at (4,3) $z = 3\times4 + 2\times3 = 18$ at (0,5) $z = 3\times0 + 2\times5 = 10$ maximum at (4,3); value is 18	2	