

ANSWER KEY

SECOND YEAR HIGHER SECONDARY EXAMINATION MARCH 2022.

PART-I/II/III

SUBJECT: Mathematics - Commerce 80

CODE NO: ~~5457~~ 5457

VERSION: S

80 SCORES

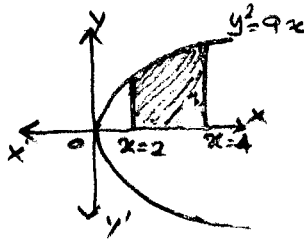
2 1/2 HOURS

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
1.		(C) $(8, 6) \in R.$	1	1
2.		(B) $\frac{11}{3}$	1	1
3.		(C) $m = n$	1	1
4.		$\int_2^6 y dx$	1	1
5.		degree = 3	1	1
6.		$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+1}{3}$	1	1
7.		(C) $k^3 \cdot  A $	1	1
8.		(B) $a^2 \cdot \log a.$	1	1
9		(A) 1, 0, 0	1	1
10		$P(E \cap F) = \frac{1}{6}$	1	1
11		$f \circ g(x) = f(g(x))$ $= f(x^{1/3}) = 8(x^{1/3})^3 = 8x.$ $g \circ f(x) = g(fx)$ $= g(8x^3) = (8x^3)^{1/3} = 2x.$	1 1	2.
12.		$R'(x) = 6x^2 + 36$ Marginal Revenue when $x=15$ $= 6 \times 15^2 + 36 = 126$	1 1	2

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
13.		$u = x^2 \quad du = 2x dx$ $\int \frac{2x}{1+x^4} dx = \int \frac{1}{1+u^2} du$ $= \tan^{-1} u + C$ $= \tan^{-1} x^2 + C.$	1 1	2
14.		$A(3, 8) \quad B(-4, 2) \quad C(5, 1)$ $\text{Area of } \triangle ABC = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $= \frac{1}{2} \begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & 1 & 1 \end{vmatrix}$ $= \frac{1}{2} [3(2-1) - 8(-4-5) + (-4-10)]$ $= \frac{1}{2} [3 + 72 - 14]$ $= \frac{61}{2} \text{ sq. units}$	1 1	2
15		$\vec{r} = \vec{a} + \lambda \vec{b} \quad \vec{a} = -i + 0j + 2k \quad \vec{b} = 3i + 4j + 6k$ $\vec{r} = (-i + 0j + 2k) + \lambda(3i + 4j + 6k)$	1 1	2
16.		$\tan^{-1} [2 \cos 2(\sin^{-1} \frac{1}{2})]$ $= \tan^{-1} [2 \cos 2 \times \frac{\pi}{6}]$ $= \tan^{-1} [2 \times \cos \frac{\pi}{3}]$ $= \tan^{-1} (2 \times \frac{1}{2})$ $= \tan^{-1} (1) = \frac{\pi}{4}$	1 1	2
17		$\text{Let } A = 1.A$ $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$ $R_2 \Rightarrow R_2 - R_1 \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} A$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$R_1 \rightarrow R_1 - R_2$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} A$ $\therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$	1	2
18		$\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx = 2 \cdot \int_0^{\pi/2} \sin^2 x \, dx$ $= 2 \cdot \int_0^{\pi/2} \frac{1 - \cos 2x}{2} \, dx$ $= \left[ x - \frac{\sin 2x}{2} \right]_0^{\pi/2}$ $= \frac{\pi}{2} - 0 = \frac{\pi}{2}$	1	2
19.	a)	$f(x) = 4x + 3$ $f(x_1) = f(x_2) \Rightarrow 4x_1 + 3 = 4x_2 + 3$ $\Rightarrow 4x_1 = 4x_2$ $\Rightarrow x_1 = x_2 \therefore f \text{ is 1-1.}$ Let $y = 4x + 3 \Rightarrow 4x = y - 3$ $x = \frac{y-3}{4} \quad y \in \mathbb{R}$ Let $\therefore f(x)$ is onto $\therefore f$ is both 1-1 and onto. Hence $f$ is bijective	1	4
	b)	Let $g(x) = \frac{x-3}{4}$ $f \circ g(x) = f(g(x)) = 4\left(\frac{x-3}{4}\right) + 3 = x$ $g \circ f(x) = g(f(x)) = \frac{4x+3-3}{4} = \frac{4x}{4} = x$ $\therefore f^{-1} = g(x) = \frac{x-3}{4}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
20	a	$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$	1	4.
	b	$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{2}{11} = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{2}{11}}{1 - \frac{1}{2} \times \frac{2}{11}}\right)$	1	
		$= \tan^{-1}\left(\frac{11+4}{22-2}\right)$	1	
		$= \tan^{-1}\left(\frac{15}{20}\right) = \tan^{-1}\frac{3}{4}$	1	
21.		$f(x) = 2x^3 - 3x^2 - 36x + 7$ $f'(x) = 6x^2 - 6x - 36$ $f'(x) = 0 \Rightarrow 6(x^2 - x - 6) = 0$ $\Rightarrow 6(x-3)(x+2) = 0$ $\Rightarrow x = 3 \quad x = -2$ $\ln(-\infty, -2) \quad f'(x) > 0$ $\ln(-2, 3) \quad f'(x) < 0$ $\ln(3, +\infty) \quad f'(x) > 0$ Strictly Increasing on $(-\infty, -2) \cup (3, +\infty)$ Strictly Decreasing on $(-2, 3)$	1 1 1	4
22.	a)	$\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left  \frac{x-a}{x+a} \right  + C$ $\therefore \int \frac{1}{x^2 - 16} dx = \frac{1}{2 \times 4} \log \left  \frac{x-4}{x+4} \right  + C$ $= \frac{1}{8} \cdot \log \left  \frac{x-4}{x+4} \right  + C$	1 1	4.
	b)	$\int_0^1 \frac{\tan^{-1}x}{1+x^2} dx$ Put $u = \tan^{-1}x$ $du = \frac{1}{1+x^2} dx$ when $x=0 \quad u=0$ $x=1 \quad u = \pi/4$ $\therefore \int_0^1 \frac{\tan^{-1}x}{1+x^2} dx = \int_0^{\pi/4} u du = \left[ \frac{u^2}{2} \right]_0^{\pi/4} = \frac{\pi^2}{32}$	1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
23	a	<p>Required Area</p> $= \int_2^4 y \, dx$ $= \int_2^4 3\sqrt{x} \, dx$ $= 3 \cdot \left[ \frac{x^{3/2}}{3/2} \right]_2^4 = 2 \cdot \left[ x^{3/2} \right]_2^4$ $= 2 \cdot (8 - 2\sqrt{2}) = 16 - 4\sqrt{2}$ 	1 1 1 1	4
24	a)	$a * b = \frac{ab}{4}$ $b * a = \frac{ba}{4} = \frac{ab}{4} = a * b$ <p><math>\therefore *</math> is commutative</p>	2	
	b)	$(a * b) * c = \left( \frac{ab}{4} \right) * c = \frac{abc}{16}$ $\Leftrightarrow a * (b * c) = a * \left( \frac{bc}{4} \right) = \frac{abc}{16}$ <p><math>\therefore (a * b) * c = a * (b * c)</math>  <math>*</math> is associative</p>	1	4
	c.	<p>Let <math>a * e = a \Rightarrow \frac{ae}{4} = a</math></p> $\Rightarrow e = 4.$ <p><math>\therefore</math> Identity element = 4.</p>	1	
25	25	<p>Let <math>\Delta = \begin{vmatrix} 1 &amp; a &amp; bc \\ 1 &amp; b &amp; ca \\ 1 &amp; c &amp; ab \end{vmatrix}</math></p> $R_2 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - R_1$ $= \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$ $= \begin{vmatrix} (b-a) & c(a-b) \\ (c-a) & b(a-c) \end{vmatrix}$	2 1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$= (a-b)(c-a) \cdot \begin{vmatrix} -1 & c \\ 1 & -b \end{vmatrix}$ $= (a-b)(c-a)(b-c)$	1	4
26		$\begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ $A \cdot X = B$ $ A  = \begin{vmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{vmatrix}$ $= 3(2-3) + 2(4+4) + 3(-6-4)$ $= -17 \neq 0.$ <p>Now <math>A^{-1} = \frac{-1}{17} \begin{bmatrix} -1 &amp; -5 &amp; -1 \\ -8 &amp; -6 &amp; 9 \\ -10 &amp; 1 &amp; 7 \end{bmatrix}</math></p> $X = A^{-1} \cdot B = \frac{-1}{17} \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix} \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix}$ $= \frac{-1}{17} \begin{bmatrix} -17 \\ -34 \\ -51 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $x = 1 \quad y = 2 \quad z = 3.$	1 1 2	6
27		<p>When <math>x &lt; 2</math> <math>f(x) = x^3 - 3</math>, a polynomial  <math>\therefore</math> Continuous when <math>x &lt; 2</math>.</p> <p>when <math>x &gt; 2</math> <math>f(x) = x^2 + 1</math>, polynomial  <math>\therefore</math> Continuous when <math>x &gt; 2</math>.</p> <p>Now <math>\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x^3 - 3 = 5</math></p> $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (x^2 + 1) = 5$ $f(5) = 5$ <p><math>\therefore \lim_{x \rightarrow 2} f(x) = f(2) = f(2) \therefore</math> Continuous at <math>x=2</math>.</p>	1 1	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
	b)	$\frac{d}{dx}(\cos \sqrt{x}) = -\sin \sqrt{x} \cdot \frac{1}{2\sqrt{x}} \times \frac{d}{dx}(\sqrt{x})$ $= -\sin \sqrt{x} \cdot \frac{1}{2\sqrt{x}}$ $= -\frac{\sin \sqrt{x}}{2\sqrt{x}}$	2	6.
	c)	$x^2 + xy + y^2 = 100$ <p>Differentiating wr to x,</p> $2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0.$ $(x + 2y) \frac{dy}{dx} = -(2x + y)$ $\frac{dy}{dx} = -\frac{(2x + y)}{(x + 2y)}$	1 1	
28.	a)	$y = mx \quad \text{--- (1)}$ $\frac{dy}{dx} = m$ <p>Using in (1), <math>y = \frac{dy}{dx} \cdot x</math>, is the required differential equation.</p>	2	
	b)	$\frac{dy}{dx} = (1+x^2) \cdot (1+y^2); \text{ variable}$ <p>Separable form.</p> $\therefore \frac{dy}{1+y^2} = (1+x^2) dx.$ <p>Integrating,</p> $\int \frac{1}{1+y^2} dy = \int (1+x^2) dx$ $\tan^{-1} y = x + \frac{x^3}{3} + C.$	4	6.

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
29	a)	$\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ $\vec{a} - \vec{b} = 0\hat{i} - \hat{j} - 2\hat{k}$	2	6.
	b)	$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 2 \times 0 + 3 \times (-1) + 4 \times (-2)$ $= 0 - 3 - 8 = -11.$	2	
	c)	$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$ $= (-6 + 4)\hat{i} - (-4 - 0)\hat{j} + (-2 - 0)\hat{k}.$ $= -2\hat{i} + 4\hat{j} - 2\hat{k}.$	2	
30.	a)	$x^y = y^x$ <p>Taking log on both sides.</p> $y \log x = x \log y$ <p>Differentiating w.r to x.</p> $y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \log y \cdot 1$ $\left[ \log x - \frac{x}{y} \right] \frac{dy}{dx} = \log y - \frac{y}{x}$ $\frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$ $= \frac{y [x \log y - y]}{x [y \log x - x]}$	3	
	b)	$y = \sin^{-1} x$ <p><math>\therefore</math> <del>Sin y</del> Differentiating w.r to x</p>		



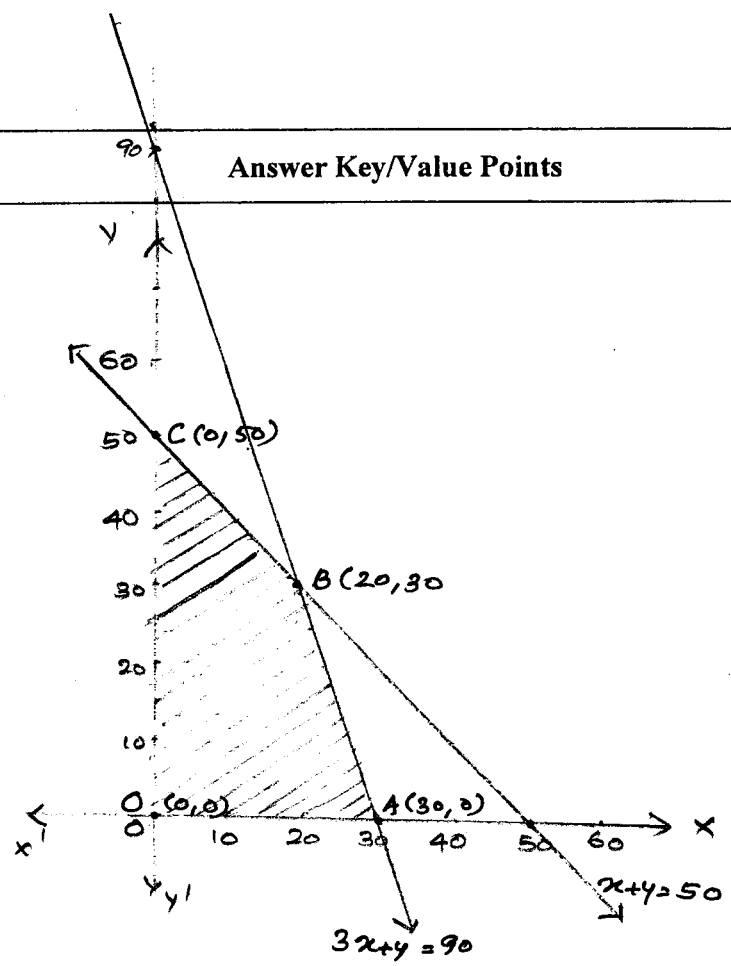
Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ $\sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$ <p>D. w.r to <math>x</math> again.</p> $\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = 0.$ $(1-x^2) \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot x^{-1} = 0.$ $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0.$	3	6.
31.	a)	<p>Let <math>y = \sqrt{x}</math> and let <math>x = 25</math> and <math>\Delta x = 0.3</math>.</p> <p>Then <math>\Delta y = \sqrt{x + \Delta x} - \sqrt{x}</math></p> $= \sqrt{25.3} - \sqrt{25} = \sqrt{25.3} - 5$ <p><math>\therefore 5 + \Delta y = \sqrt{25.3}</math>.</p> <p>Now <math>\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}</math> or <math>\Delta y \approx \left(\frac{dy}{dx}\right) \cdot \Delta x</math></p> $\therefore \sqrt{25.3} \approx 5 + \frac{dy}{dx} \cdot \Delta x$ $\approx 5 + \frac{1}{2\sqrt{x}} \cdot \Delta x$ $\approx 5 + \frac{1}{2\sqrt{25}} \cdot 0.3$ $\approx 5 + 0.03$ $\approx 5.03$	3	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
	b).	$P(x) = 41 + 72x - 18x^2$ $P'(x) = +72 - 36x$ $P''(x) = -36 < 0.$ <p>Now <math>P'(x) = 0 \Rightarrow 36x = +72</math>  <math>x = +2.</math></p> <p>at <math>x = +2</math> <math>P''(x) &lt; 0</math>  <math>\therefore P(x)</math> is Maximum when <math>x = 2</math></p> <p><math>\therefore</math> Maximum profit = <math>P(2)</math>  <math>= 41 + 72 \times 2 - 18 \times (2)^2</math>  <math>= 113</math> units</p>	3	6.
32.	a)	$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{ \vec{b}_1  \cdot  \vec{b}_2 }$ $= \frac{2 - 1 + 2}{\sqrt{1+1+1} \sqrt{4+1+4}}$ $= \frac{3}{\sqrt{3} \cdot \sqrt{9}} = \frac{1}{\sqrt{3}}$ $\Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right).$	2	6.
	b)	$SD = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 0\hat{j} + 3\hat{k}$ $SD = \frac{ (\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-3\hat{i} + 0\hat{j} + 3\hat{k}) }{\sqrt{9+0+9}}$ $= \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}.$	4	

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
33.	a)	$A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} \quad a_{ij} = 2i - j$	2	
	b)	$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 4 \\ -2 & -1 & 3 \end{bmatrix} \quad A' = \begin{bmatrix} 6 & -2 & -2 \\ -2 & 3 & -1 \\ 2 & 4 & 3 \end{bmatrix}$ <p>Let <math>P = \frac{1}{2} (A + A')</math></p> $= \frac{1}{2} \begin{bmatrix} 12 & -4 & 0 \\ -4 & 6 & 3 \\ 0 & 3 & 6 \end{bmatrix}$ <p>Let <math>Q = \frac{1}{2} (A - A')</math></p> $= \frac{1}{2} \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$ <p>Now <math>P' = P</math> and <math>\therefore P</math> is Symmetric</p> <p><math>Q' = -Q \quad \therefore Q</math> is Skew symmetric</p> <p>Also</p> $P + Q = \frac{1}{2} \begin{bmatrix} 12 & -4 & 0 \\ -4 & 6 & 3 \\ 0 & 3 & 6 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & 0 & 4 \\ 0 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$ $= \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 4 \\ -2 & -1 & 3 \end{bmatrix}$ <p><math>= A.</math></p> <p><math>\therefore A</math> is the Sum of <math>P</math> and <math>Q</math> where <math>P</math> is Symmetric and <math>Q</math> is skew Symmetric.</p>	6	8

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
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34.



Shaded region is the feasible region

Corner Points	Value of Z.
(0,0)	$Z = 0$
(30,0)	$Z = 120$
(20,30)	$Z = 110$
(0,50)	$Z = 50$

Z is Maximum when  $x = 30, y = 0$ .  
and the maximum value of Z is 120.

8

8

35

a)

$$\begin{aligned}
 P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\
 &= \frac{7}{13} + \frac{9}{13} - \frac{4}{13} \\
 &= \frac{12}{13}
 \end{aligned}$$

Qn. No	Sub Qns	Answer Key/Value Points	Score	Total Score
		$P(A B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{4/13}{9/13} = \frac{4}{9}$	3	
	b).	$\sum_{i=1}^n P(k) = 1 \Rightarrow 0.1 + k + 2k + 2k + k = 1$ $\Rightarrow k = 0.15$ $P(X < 3) = P(X=0) + P(X=1) + P(X=2)$ $= 0.1 + k + 2k.$ $= 0.1 + 3k$ $= 0.1 + 3 \times 0.15$ $= 0.55$ $P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$ $= 2k + 2k + k$ $= 5k$ $= 5 \times 0.15$ $= 0.75$ <hr/>	5	8